Teager-Huang Analysis
Applied to Sonar Target Recognition

J.C. Cexus, and A.O. Boudraa

Abstract—A new approach for target recognition based on the Empirical mode decomposition (EMD) algorithm of Huang et al. [11] and the energy tracking operator of Teager [13]-[14] are introduced. The conjunction of these two methods is called Teager-Huang analysis. This approach is well suited for nonstationary signals analysis. The impulse response (IR) of target is first band pass filtered into sub-signals (components) called Intrinsic mode functions (IMFs) with well defined Instantaneous frequency (IF) and Instantaneous amplitude (IA). Each IMF is a zero-mean AM-FM component. In second step, the energy of each IMF is tracked using the Teager energy operator (TEO). IF and IA, useful to describe the time-varying characteristics of the signal, are estimated using the energy separation algorithm (ESA) algorithm of Maragos et al. [16]-[17]. In third step, a set of features such as skewness and kurtosis are extracted from the IF, IA and IMF energy functions. The Teager-Huang analysis is tested on set of synthetic IRs of Sonar targets with different physical characteristics (density, velocity, shape, …). Principal component analysis (PCA) is applied to features to discriminate between manufactured and natural targets. The manufactured patterns are classified into spheres and cylinders. One hundred percent of correct recognition is achieved with twenty three echoes where sixteen IRs, used for training, are free noise and seven IRs, used for testing phase, are corrupted with white Gaussian noise.

Keywords—Target recognition, Empirical mode decomposition, Teager-Kaiser energy operator, Features extraction.

I. INTRODUCTION

The use of underwater sound for the purpose of detecting and locating submerged targets was introduced more than 80 years ago. The problem of discrimination of immersed targets was initiated with the works of Hoffman [1] who investigated time-domain approaches and Chesnut and Floyd who tested multiple frequency based techniques [2]. Time-domain techniques based on neural network inversions have been developed to discriminate Sonar objects [3], [4]. Time-frequency approaches have also been used for target classification [5], [6] and have given high potentiality for discrimination between solid and hollow targets as well as for determining the target material [7]. For example in [7] a Wigner-Ville distribution (WVD) [8] as a time-frequency description is used. Indeed, WVD has been shown to be relevant for understanding of echo formation mechanisms and for surface waves that circumnavigate the targets [5], [6]. In [9] a sonar target classification approach based on the time-frequency projection filtering, proposed by Hlawatsch and Kozek [10], is presented. The WVD associated to the Impulse response (IR) (acoustic response) of a Sonar target generates a time-frequency plane (image) showing different patterns. These patterns can be classified into two categories 1) Interferences due to the bilinear nature of the WVD [8]. 2) High energy pattern: the first one, non dispersive, is associated with the specular echo on the target and the two following patterns correspond to the arrival of surfaces waves (antisymmetric Lamb waves) that circumnavigate the target [9]. The two pertinent patterns for classification are the specular reflection and the Lamb waves. The function of a time-frequency filter is to extract from the signal to be analyzed the pertinent patterns. The filter is designed from the WVD of a reference signal and more particularly from its time-frequency support \( R \) containing the relevant information. This region \( R \) is derived manually (isolation of the echoes by an expert operator). The difficult with the WVD is the severe cross terms as indicated by the existence of negative power for some frequency ranges. Although most of these difficulties are overcome by using proper kernel functions, the method is still Fourier based; therefore all the possible complications associated with Fourier transform still exist. To circumvent this difficulty we investigate the Empirical mode decomposition (EMD) method proposed by Huang et al. [11] to analyze nonlinear and non-stationary signals such as the IR of Sonar target. Indeed, the EMD does not make any assumption about the stationnarity or the non-linearity of the analyzed signals, and avoids the interference problem of the WVD. The EMD decompose a signal into intrinsic oscillatory mode. The aim is to determine the intrinsic modes which characterize the Sonar target. In practice, the EMD is easier and less complicate for implementation than WVD.

II. IMPULSE RESPONSE OF THE TARGET

The IR contains most of the information available on the target. This response can be decomposed into several elementary components related to various physical phenomena such as specular echo or surface acoustic waves. These components are generally excited in different frequency ranges. The acoustic field scattered by a target is calculated using a decomposition of the backscattered pressure field into an infinite summation of modal components, depending on the mechanical properties and the geometry of the scatter [12]. The modal components are the eigenfunctions of the spherical

A.O. Boudraa, and J.C. Cexus are with IRENav, Ecole navale, Lanvéoc-Poulmic, BP 600, 29240, Brest-Armées, France (corresponding author: A.O. Boudraa, e-mail: boudraa@ecole-navale.fr).
or cylindrical target. The IR of the target is computed from the scattered pressure spectrum by taking the inverse Fourier transform. Although the IR contains all the information on the scatter, some fundamental features of the circumferential waves, such as velocity dispersions, are not clearly displayed in a single time or single frequency domain [7]. The use of joint time-frequency approach can thus provide significant insight for the purpose analysis.

III. SONAR TARGET RECOGNITION METHOD

A. Empirical Mode Decomposition

The principle of the EMD is to decompose adaptively a given signal \( x(t) \) into oscillating components. These components are called Intrinsic Mode Functions (IMFs) and are obtained from the signal \( x(t) \) by means of an algorithm called sifting. The name IMF is adapted because it represents the oscillation mode imbedded in the data. With this definition, the IMF in each cycle, defined by the zero crossings of \( x(t) \), involves only one mode of oscillation, no complex riding waves are allowed. Thus, an IMF is not extracted to a narrow band signal, and it can be both amplitude and frequency modulated. In fact, it can be non-stationary. The essence of the EMD is to identify the IMF by characteristic time scales, which can be defined locally by the time lapse between two extrema of an oscillatory mode or by the time lapse between two zero crossings of such mode. The EMD picks out the highest frequency oscillation that remains the signal. Thus, locally, each IMF contains lower frequency oscillations than the one extracted just before. Furthermore, the EMD does not use any pre-determined filter or wavelet function. It is fully data driven method. It has been shown experimentally that the EMD acts essentially as a dyadic filter bank resembling those involved in wavelet decomposition [13]. Since the decomposition of the EMD is a decomposition of the EMD is to identify the IMF by characteristic time scales, which can be defined locally by the time lapse between two extrema of an oscillatory mode or by the time lapse between two zero crossings of such mode. The EMD decomposes \( x(t) \) into oscillating components. These components are called Intrinsic Mode Functions (IMFs) and are obtained from the signal \( x(t) \) by means of an algorithm called sifting. The name IMF is adapted because it represents the oscillation mode imbedded in the data. 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\begin{align*}
&\text{Step1) Initialize: } t_0(t) \leftarrow x(t), i \leftarrow 1 \\
&\text{Step2) Extract the \text{-th IMF} } \hspace{1cm} \\
&a) \text{ Initialize } h_0(t) \leftarrow t_0(t), i \leftarrow 1 \\
&b) \text{ Extract the local minima and maxima of } h_{j-1}(t) \\
&c) \text{ Interpolate the local maxima and local minima by a cubic spline to form upper and lower envelopes of } h_{j-1}(t) \\
&d) \text{ Calculate the mean } \mu_{j-1}(t) \text{ of the upper and lower envelopes} \\
&e) \text{ } h_j(t) \leftarrow h_{j-1}(t) - \mu_{j-1}(t) \\
&f) \text{ if stopping criterion is satisfied } \\
&\hspace{1cm} \text{then IMF}_j(t) \leftarrow h_j(t) \\
&\hspace{1cm} \text{else } \{ j \leftarrow j+1 \text{ and goto (b)} \} \\
&\text{Step3) } r_j(t) \leftarrow r_{j-1}(t) - \text{IMF}_j(t) \\
&\text{Step4) if } r_j(t) \text{ still has at least 2 extrema } \\
&\hspace{1cm} \text{then } \{ j \leftarrow j+1 \text{ and goto (b)} \} \\
&\hspace{1cm} \text{else the decomposition is finished} \\
&\hspace{1cm} \text{and } r_j(t) \text{ is the residue} \\
\end{align*}

At the end of the sifting process, we have

\[ x(t) = \sum_{i=1}^{n} \text{IMF}_i(t) + r_n(t) \]

where \( \leftarrow \) denotes the affection operation and \( n \) is the number of IMFs and \( r_n(t) \) is the residue of the decomposition. If the residue has almost a zero value, this means that \( x(t) \) had a zero mean. More generally, the residue captures the trend of \( x(t) \).

B. Teager-Huang analysis

The EMD method is not itself a time-frequency representation as WVD. With the Hilbert transform, the IMF yields IFs as functions of time that give sharp identifications of imbedded structures. The final presentation of the results is an energy-frequency-time representation, designated as Hilbert-Huang transform [11]. To generate the IF and the IA of \( x(t) \) the EMD is combined with the TEO [14]-[16]. The conjunction of the EMD and the TEO methods is designated as Teager-Huang analysis (THA). More particularly, the Energy separation algorithm (ESA) [17]-[18] uses the TEO to estimate the IA and the IF of \( x(t) \). The ESA is very simple demodulating technique yielding very small errors for AM-FM demodulation. It is less computationally complex and has better time resolution than other classical demodulation approaches such as the Hilbert transform [19]. Note that the EMD can also be combined with the AM-FM demodulation technique [20].

C. TEO algorithm

It is shown that the TEO can track the energy of a signal and identify the IA and the IF [17]. The TEO, \( \Psi \), is defined for continuous-time signal \( x(t) \) as:

\[ \Psi \left[ x(t) \right] = \left[ x'(t)^2 - x(t)x''(t) \right] \]

where \( x'(t) \) and \( x''(t) \) are the first and the second time derivatives of \( x(t) \) respectively. In the discrete case, the time
derivatives may be approximated by time differences. The discrete-time counterpart of the TEO becomes [17]:

\[ \Psi'[x(n)] = x[n]^2 - x[n+1]x[n-1] \]

An important aspect of the TEO is that it is nearly instantaneous. This is because only three samples are required for the energy computation at each time instant. Furthermore, this operator is very easy to implement efficiently. The ESA [17]-[18] uses the TEO to separate into its amplitude envelope (IA) \( |a(t)| \) and IF signal \( f(t) \) to accomplish monocomponent AM-FM signal demodulation:

\[ f(t) \approx \frac{1}{2\pi} \sqrt{\frac{\Psi'[x(t)]}{\Psi'[x(t)]}} \]

\[ |a(t)| \approx \sqrt{\frac{\Psi'[x(t)]}{\Psi'[x(t)]}} \]

Since the speech signal is composed of superposition of AM-FM signals, the TEO has been successfully used in various speech processing applications [18],[21]. The main disadvantage of this operator is a moderate sensitivity to noise. Furthermore, it assumes that the estimated IF does not vary too fast (small bandwidths) or too greatly compared with the carrier frequency. The TEO is typically applied to a bandpass signal. If \( x(t) \) is a multicomponent AM-FM signal, then bandpass filtering is needed to isolate each component before applying the ESA. In this paper EMD is used as a multiband filtering to separate the components in the temporal domain and hence reduce multicomponent demodulation to monocomponent one [22].

D. Pseudo-Code of the Method

The Target recognition strategy involves the following steps:

**Input:** Signal \( x(t) \) //Target IR

1. Apply EMD to \( x(t) \).
2. Select a subset of IMFs (\( N_s \) is the cardinal of this set).
3. Apply TEO to selected IMFs, \( c_k(t) \) (k ∈ \{1,2,...,\( N_s \)}).
4. Demodulate each IMF using the TEO to estimate the IA \( a_k(t) \) and the IF \( f_k(t) \).
5. Features extraction: For \( a_k(t) \), \( f_k(t) \) and \( c_k(t) \) calculate the attributes:
   - Skewness, Kurtosis and Shannon’s Entropy.
6. Classification using extracted features (PCA,...)

**Output:** Features //Attributes for classification

The method is also described by the following diagram:

Block diagram of the THA applied to Sonar target recognition.

IV. RESULTS

The THA is tested on simulated discrete-time IRs of Sonar targets [23]. The aim is the discrimination between a Shell target (S: man-made)/Non-Shell (NS) target. Sixteen RIs for the training phase are used (Table I). For the testing phase we use seven RIs, whose characteristics are unknown, and are not used during the training phase (Table II). These seven RIs are corrupted with white Gaussian noise with signal to noise ratio \( \text{SNR}=18 \text{ dB} \). Note that Six NS man-made targets (1-3, 9-11) are used for training and four NS, natural targets (17,18,21,22) for testing. Figure 1 shows the sequential extraction into local oscillations by the EMD of an IR (shell sphere). Only a few IMFs (IMF 1, 2 and IMF n and residue \( r_n(t) \)) are shown. Remark that the first IMF corresponds to fast oscillation while the last one to slow one. There is no criteria to select the number of IMFs used in features extraction. However, the processing of a large number of IRs has shown that the more discriminant features are derived from the first IMFs. In this paper \( N_s \) is set to 4. Figure 2 shows the clustering results in training phase using PCA. For better illustration, clustering results are represented in 3D with three principal directions 2, 3 and 4. We observe a good separation in the features space, in particular for the cylinder shell and the other echoes. The rate of recognition is about 100%. Figure 2 shows also the good identification of targets (S or NS) and the recognition targets (sphere or cylinder) in the testing phase. Note that even no natural targets are used in training, they are well recognized in testing step. If the targets are natural (non shell) the algorithm of classification is not able to identify the shape of the target (sphere or cylinder). However, the identification of shell targets independently from the shape remains a good piece of information for military applications.

V. CONCLUSION

In this paper THA is used to estimate the IF and the IA of acoustic echoes of Sonar targets followed by a features
extraction and classification. The EMD is used as multiband filtering to separate the components in the temporal domain. Both the EMD and the TEO have a low computation complexity. The EMD is well adapted to non-stationary signals and the TEO has an excellent time resolution (operates on a few samples moving window). The THA gives good estimate of the IF and IA. It yields small errors of AM-FM demodulation. Processing of a large number of signals and comparisons to exiting methods such as VWD are necessary to show the robustness and the effectiveness of this analysis. The automatic classification of underwater signals obtained from active sonar is considered as a complex problem because of the large variability in both time and spectral characteristics in signal even obtained from the same targets. Presented results show that a small number of attributes are sufficient to identify the characteristics of the simulated Sonar target IRs with good accuracy. In addition, it is shown that the recognition problem can be viewed as a linearly separable one. It is obvious that this method must be now validated on real signals (Reverberation, Doppler, ...). In addition, a more detailed study on the relevance and the selection of the attributes is necessary.

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REFERENCES


Jean Christophe CEXUS was born in Hyeres, France. He received the Engineering degree and the M.Sc degree in control from the Ecole Supérieure des Sciences Appliquées pour l’Ingénieur de Mulhouse (ESSAIM) in control. He is currently pursuing the PhD degree in signal processing at Institut de Recherche de l’Ecole navale (IRENav), Brest-Armées, France. His research interests include time-frequency analysis, Sonar target recognition and neural networks.

Abdel-Ouahab BOUDRAA was born in Constantine, Algeria. He received the BS degree in Physics (Electronics Engineering) from Constantine Institute of Physics, University of Constantine, Algeria. He received the M.Sc in Biomedical Engineering from INSA, Lyon, the University degree in Nuclear Magnetic Resonance, the PhD degree in Image Processing and the University degrees in Statistics and Modeling and in Positron Emission Tomography all from the University of Claude Bernard, Lyon 1, France. He is currently Associate Professor of electrical engineering at Ecole navale, Brest, France. His current research interests include computer vision, vector quantization, data structures and analysis, data fusion, time frequency analysis, hard and fuzzy pattern recognition and applications of fuzzy set theory to medical image. Dr. BOUDRAA is recipient of 2003 Varian Prize awarded by the Swiss Society of Radiobiology and Medical Physics for the best published paper impacting Radiation Oncology. Dr. BOUDRAA is member of IEEE society.

Figure 1: An example, of selected, IMFs of the IR of a shell sphere.
Table I
Characteristics of the targets used in training phase.

<table>
<thead>
<tr>
<th>Nº of targets</th>
<th>Non shell (NS)</th>
<th>Shell (S)</th>
<th>Cylinders (Cy)</th>
<th>Spheres (Sp)</th>
<th>Label of the class (trained)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 (NiMol)</td>
<td>NS</td>
<td>Sp</td>
<td></td>
<td></td>
<td>■</td>
</tr>
<tr>
<td>2 (Inox)</td>
<td>NS</td>
<td>Sp</td>
<td></td>
<td></td>
<td>■</td>
</tr>
<tr>
<td>3 (Alu)</td>
<td>NS</td>
<td>Sp</td>
<td></td>
<td></td>
<td>■</td>
</tr>
<tr>
<td>4 (Pvc)</td>
<td>S</td>
<td>Sp</td>
<td></td>
<td></td>
<td>●</td>
</tr>
<tr>
<td>5 (Nylon)</td>
<td>S</td>
<td>Sp</td>
<td></td>
<td></td>
<td>●</td>
</tr>
<tr>
<td>6 (NiMol)</td>
<td>S</td>
<td>Sp</td>
<td></td>
<td></td>
<td>●</td>
</tr>
<tr>
<td>7 (Inox)</td>
<td>S</td>
<td>Sp</td>
<td></td>
<td></td>
<td>●</td>
</tr>
<tr>
<td>8 (Alu)</td>
<td>S</td>
<td>Sp</td>
<td></td>
<td></td>
<td>●</td>
</tr>
<tr>
<td>9 (NiMol)</td>
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<td>Cy</td>
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<tr>
<td>10 (Inox)</td>
<td>NS</td>
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<td>■</td>
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<td>11 (Alu)</td>
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<td>■</td>
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<tr>
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<tr>
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<td>▲</td>
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<tr>
<td>15 (Inox)</td>
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<td>Cy</td>
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<td></td>
<td>▲</td>
</tr>
<tr>
<td>16 (Alu)</td>
<td>S</td>
<td>Cy</td>
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<td>▲</td>
</tr>
</tbody>
</table>

Table II
Characteristics of the targets used in testing phase.

<table>
<thead>
<tr>
<th>Nº of targets</th>
<th>Non shell (NS)</th>
<th>Shell (S)</th>
<th>Cylinders (Cy)</th>
<th>Spheres (Sp)</th>
<th>Label of the class (desired)</th>
</tr>
</thead>
<tbody>
<tr>
<td>17 (Granit)</td>
<td>NS</td>
<td>Sp</td>
<td></td>
<td></td>
<td>□</td>
</tr>
<tr>
<td>18 (Marbre)</td>
<td>NS</td>
<td>Sp</td>
<td></td>
<td></td>
<td>□</td>
</tr>
<tr>
<td>19 (Pvc)</td>
<td>S</td>
<td>Sp</td>
<td></td>
<td></td>
<td>□</td>
</tr>
<tr>
<td>20 (Alu)</td>
<td>S</td>
<td>Sp</td>
<td></td>
<td></td>
<td>□</td>
</tr>
<tr>
<td>21 (Granit)</td>
<td>NS</td>
<td>Cy</td>
<td></td>
<td></td>
<td>□</td>
</tr>
<tr>
<td>22 (Marbre)</td>
<td>NS</td>
<td>Cy</td>
<td></td>
<td></td>
<td>□</td>
</tr>
<tr>
<td>23 (Pvc)</td>
<td>S</td>
<td>Cy</td>
<td></td>
<td></td>
<td>△</td>
</tr>
</tbody>
</table>

Figure 2: PCA results classification (3D projection on direction 2 3 4), training (full forms) and testing (shell forms) phases.