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Lateral Torsional Buckling Resistance of Trapezoidally Corrugated Web Girders

Annamária Käferné Rácz, Bence Jáger, Balázs Kövesdi, László Dunai

Abstract**—**Due to the numerous advantages of steel corrugated web girders, its application field is growing for bridges as well as for buildings. The global stability behavior of such girders is significantly larger than those of conventional I-girders with flat web, thus the application of the structural steel material can be significantly reduced. Design codes and specifications do not provide clear and complete rules or recommendations for the determination of the lateral torsional buckling (LTB) resistance of corrugated web girders. Therefore, the authors made a thorough investigation regarding the LTB resistance of the corrugated web girders. Finite element (FE) simulations have been performed to develop new design formulas for the determination of the LTB resistance of trapezoidally corrugated web girders. FE model is developed considering geometrical and material nonlinear analysis using equivalent geometric imperfections (GMNI analysis). The equivalent geometric imperfections involve the initial geometric imperfections and residual stresses coming from rolling, welding and flame cutting. Imperfection sensitivity analysis was performed to determine the necessary magnitudes regarding only the first eigenmodes shape imperfections. By the help of the validated FE model, an extended parametric study is carried out to investigate the LTB resistance for different trapezoidal corrugation profiles. First, the critical moment of a specific girder was calculated by FE model. The critical moments from the FE calculations are compared to the previous analytical calculation proposals. Then, nonlinear analysis was carried out to determine the ultimate resistance. Due to the numerical investigations, new proposals are developed for the determination of the LTB resistance of trapezoidally corrugated web girders through a modification factor on the design method related to the conventional flat web girders.

Keywords**—**Critical moment, FE modeling, lateral torsional buckling, trapezoidally corrugated web girders.

I. Introduction

THE application of trapezoidally corrugated web girders (TWGs) is increasing due to the favorable structural behavior. Previous researches have shown that it may have a higher resistance to LTB than girders with flat webs, however, the current codes do not have any recommendations to calculate the LTB resistance of TWGs [1]. Previous papers give recommendations for the modification of the existing formulas developed for flat web girders. Most of them suggest altering one or more cross-sectional property of the girder, such as the warping constant, the shear modulus, or using an equivalent thickness for the web.

The purpose of the current research is to study the relationship between the LTB resistance of trapezoidally corrugated and flat web girders having the same web thickness. The critical moment of these girders is compared and an enhanced simplified modification factor is introduced describing the difference between the critical moment of trapezoidally corrugated and flat web girders subjected to bending. The used notations in the paper are shown in Fig. 1.

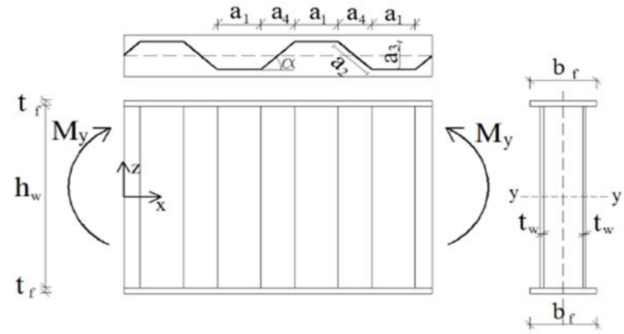

Fig. 1 Geometry of the girder

II. Previous Studies

The previous experimental and numerical investigations are collected and evaluated. Papers dealing with TWGs and sinusoidally corrugated web girders are also included. Several papers give proposals on how to take the effect of the trapezoidally corrugated web into account coupled with existing formulas developed for flat web girders.

Lindner [2] was the first person who executed experiments in this research field. His suggestion was to modify the warping constant according to (1).

$$I_w^{co} = I_{w,flat} + c_w \cdot \frac{L^2}{E \cdot \pi^2}, \tag{1}$$

where $c_w = \frac{a_3^2 \cdot h_m^2}{8 \cdot u_x \cdot (a_1 + a_4)}$, $I_{w,flat}$ the warping constant of the flat web girder, h_m is the total height of the girder, L is the length of the girder, G is the shear modulus,

$$u_x = \frac{h_m}{2 \cdot G \cdot a_1 \cdot t_w} + \frac{h_m^2 \cdot (a_1 + a_4)^3 \cdot (I_{y1} + I_{y2})}{600 \cdot a_1^2 \cdot E \cdot (I_{y1} \cdot I_{y2})},$$

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E is the elastic modulus (Young's modulus), I_{y1} and I_{y2} are the inertia of the flanges.

Sayed-Ahmed [3] recommended to take the trapezoidally corrugated web into account by applying an equivalent thickness of the web in the critical moment calculation. The equivalent thickness may be calculated by (2).

$$t_{eq} = \frac{a_1 + a_2}{a_1 + a_4} \cdot t_w \,. \tag{2}$$

Using Lindner's [2] experimental results, Moon et al. [4] introduced a new method for calculating the warping constant. The recommendation based on the fact, that along the girder the shear center changes. Equation (3) shows the average distance between the corrugation and the centroid of the flanges proposed by Moon et al. [4]. Then, the warping constant is calculated according to (4).

$$d_{avg} = \frac{(2a_1 + a_4)}{2(a_1 + a_4)} d_{\max} \,. \tag{3}$$

$$I_{w,co} = \frac{1}{3} \sum (W_{ni}{}^2 + W_{nj} W_{ni} + W_{nj}{}^2) \cdot t_{ij} L_{ij} \,. \tag{4}$$

where W_i is the normalized unit warping at point i, d_{max} is the maximum deviation of the web form the centroid of the flanges, t_{ij} is the thickness of the plate between i and j, L_{ij} is the distance of i and j.

Apart from the warping constant the shear modulus was suggested to be calculated in a different way as well. The formula introduced by Johnson and Cafolla [5] is shown in (5), where the G_{flat} represents the shear modulus of the flat web.

$$G_{co} = \frac{(a_1 + a_4)}{(a_1 + a_2)} \cdot G_{flat} \,. \tag{5}$$

Nguyen et al. [6] described the distance between the shear center and the centroid according to (6) and from that the warping constant can be calculated as shown in (7).

$$X_s = d + \frac{6 b_f t_f}{6 b_f t_f + h_w t_w} d \,. \tag{6}$$

$$I_{w,co} = \frac{h_w^2 t_f b_f (6 t_f b_f^3 + t_w h_w b_f^2 + 12 d^2 t_w h_w)}{24(6 b_f t_f + h_w t_w)} \tag{7}$$

where d is the distance between the corrugation and the centroid of the flanges.

Zhang et al. [7] considered the trapezoidally corrugated web as an eccentric flat web. The warping constant is calculated according to (8).

$$I_{w,co} = I_{w,flat} + \frac{t_w h_w^3 e^2}{12} \,. \tag{8}$$

where $e^2 = (\frac{a_3}{2})^2 \frac{a_1 + \frac{a_4}{3}}{2q}$ is the eccentricity, and q is the wavelength.

Larsson and Persson [8] suggested to modify the torsional constant as well, along with the warping constant. Their suggestion for the calculation is shown in (9) and (10).

$$I_t^e = \frac{Q}{\theta_L G}(L - a) \,. \tag{9}$$

$$I_\omega^e = \frac{G I_t}{E} a^2 \,. \tag{10}$$

where $a = \sqrt{\frac{E I_\omega}{G I_t}}$, Θ_L is the rotation of the end cross-section, Q is the applied external torsional moment.

Ilanovsky [9] introduced a modification factor for C_1 according to (11), which factor can be used without any modification of the cross-sectional properties.

$$C_{cw,1} = C_1 + \frac{a_3 \cdot L}{8} \,. \tag{11}$$

Most of the recommendations modify one or more cross-sectional properties of the I-girder. The alteration of the warping constant is the most common in the international literature, however, the equivalent thickness and the modification of the torsional constant are proposed as well. The results and conclusions of these studies gave a basis for the investigation and for the development of the numerical model.

III. Numerical Model Development

An advanced numerical model is developed in ANSYS 18.2 [10] finite element program. It is based on a full shell model using four-node-thin shell elements. Fig. 2 presents the geometrical model with the boundary and loading conditions. The bending moment is substituted by a force-pair applied in the center of gravities of the upper and lower flanges. The bending moment is permanent along the girder length in the current investigations.

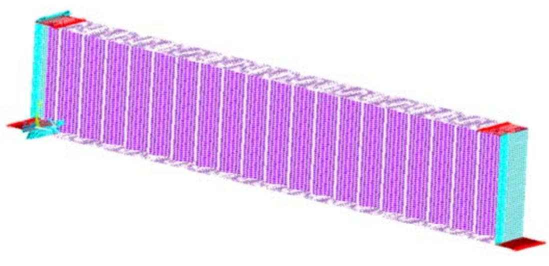

Fig. 2 Boundary and loading conditions

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A. Applied Analysis and Material Model

Two analysis types are applied, (i) linear buckling analysis (LBA) and (ii) geometrical and material nonlinear analysis using equivalent geometric imperfections (GMNIA). A linear elastic - hardening plastic material model with von Mises yield criterion is used in the numerical model. The material model behaves linear elastic up to the yield strength (f_y) by obeying Hook's law with Young's modulus equal to 210000 MPa. The yield plateau is modeled up to 1% strains with a small increase in the stresses. By exceeding the yield strength, the material model has an isotropic hardening behavior with a hardening modulus until it reaches the ultimate strength (f_u). From this point, the material is assumed to behave as perfectly plastic.

B. Geometric Parameters

Eight different cross-sections with an average of nine different lengths are studied in the executed numerical research program. Table I shows the eight different cross-section types to be analyzed. The corrugation folds of the trapezoidally corrugated web have the same length (a_1=a_2) for all the analyzed geometries.

TABLE I
CROSS-SECTION TYPES

No.	t_f (mm)	b_f (mm)	t_w (mm)	h_w (mm)	a_1 (mm)	α (°)
T1	20	250	6	1000	150	30
T2	20	250	6	1000	150	45
T3	20	350	6	1000	250	30
T4	20	350	6	1000	250	45
T5	20	200	4	600	150	30
T6	30	350	4	600	250	45
T7	30	350	6	1000	250	30
T8	40	350	8	1300	250	45

C. Applied Imperfections

In the GMNIA, the first eigenmode shapes are applied as equivalent geometric imperfections. The value of the maximal imperfection magnitude is L/200 proposed by EN1993-1-1 [11]. A typical LTB eigenmode shape is presented in Fig. 3.

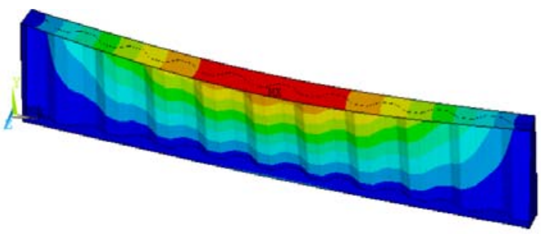

Fig. 3 Eigenmode shape

To be able to compare the LTB resistance, the same cross-section types with conventional flat web girders are also analyzed as references. The length of the analyzed girders is between 2.8-51.2 m.

D. Model Verification

The FE model is validated based on the previous experimental test results of Kubo and Watanabe [12]. The experiments were carried out with simply supported girders subjected to three-point-bending. The boundary conditions are the same as in the original model. In Table II, the geometric parameters of the tested girders are introduced.

TABLE II
GEOMETRIC PARAMETERS FOR VALIDATION

Specimen ID	t_f (mm)	b_f (mm)	t_w (mm)	h_w (mm)	a_1 (mm)	α (°)	L (m)	f_y (MPa)
CB150-6	8.25	100.3	3.26	319.2			1.5	
CB210-6	8.35	100.8	3.18	319.4	87	43.6	2.1	297
CB270-6	8.10	100.2	3.29	320.2			2.7	

The ultimate strengths of the girders are presented in Table III, where P_u is the experimental and $P_{u,num}$ is the computed ultimate resistance. The applied imperfection magnitude is L/200 for all girders as described in section *C*. The results show that the numerical model provides safe side resistances.

TABLE III
ULTIMATE STRENGTH FOR VALIDATION

Specimen ID	P_u (kN)	$P_{u,FEM}$ (kN)	Error
CB150-6	193.8	161.7	17%
CB210-6	113.6	100.8	11%
CB270-6	93.7	64.7	31%

During validation, an imperfection-sensitivity analysis is also carried out. Fig. 4 shows one of the sensitivity curves. The ultimate strength from the numerical model with different imperfection magnitudes is represented by $P_{u,FEM}$, and the ultimate strength from measured in the tests is given by P_u.

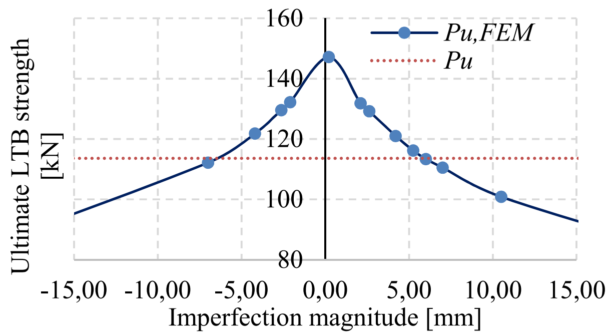

Fig. 4 Imperfection sensitivity analysis

The character of the imperfection sensitivity curves is symmetric as shown in Fig. 4. The necessary imperfection magnitudes – represented by the intersection point of the horizontal line and the computed blue curve – are always smaller than the proposal of the EN1993-1-1 [10]. Thus, adequate safe side solutions may be provided by the standard which is judged to be acceptable for further use in the parametric study.

IV. RESULTS

A. Failure Modes

The failure modes of the analyzed girders are different. Especially as the length of the girders decreases, local flange buckling can occur instead of the global stability failure, LTB. To avoid the local flange buckling failure mode, the computed first eigenmodes of all the studied girders have been studied before the carrying out a nonlinear analysis. A typical LTB failure mode is presented in Fig. 5.

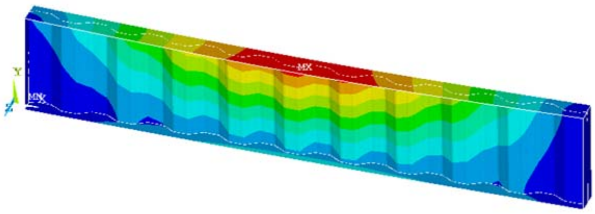

Fig. 5 LTB failure mode

B. Numerical Parametric Study

The aim of the numerical parametric study is to determine the relationship and reserve in the LTB resistance of the TWGs compared to the flat web girders, therefore a new modification factor (C_{tr}) is developed. The modification factor is introduced in the form of (12). This modification factor is meant to consider the calculated difference in the LTB behavior of TWGs compared to flat web girders.

$$M_{cr,num,tr} = C_{tr} \cdot M_{cr,num,flat} \,. \tag{12}$$

where $M_{cr,num,tr}$ is the numerically computed critical moment for TWGs, C_{tr} is the modification factor, and $M_{cr,num,flat}$ is the numerically computed critical moment for flat web girders.

For the investigated girders, the C_{tr} modification factor is determined by curve fitting using the FE analysis, which results are shown in Fig. 6. The horizontal axis shows the length of the girder, which is the same as the buckling length due to the simply supported boundary conditions allowing the rotation at both supports.

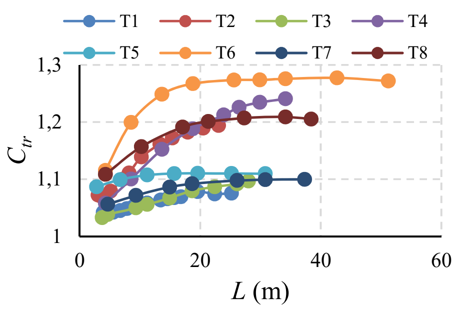

Fig. 6 Values of C_{tr}

C. Design Model Development

It can be observed that the curves start from 1.0 and they could be approximated by square root functions. It is also shown that the critical moment of the TWG is always obtained higher LTB strength equivalent girders with flat web. According to these observations, the modification factor, C_{tr}, is developed in the form of (13).

$$C_{tr} = 1.0 + \sqrt{m \cdot L} \,. \tag{13}$$

Fig. 7 shows the approximation of specimens T1-T4. The vertical and horizontal axes represent the ($m \cdot L$) value and the girder length (L), respectively. It can be observed that the gradient of the provided curves can be approximated by straight lines with inclinations of m. The approximated values of m for all the analyzed girder geometries are summarized in Table IV.

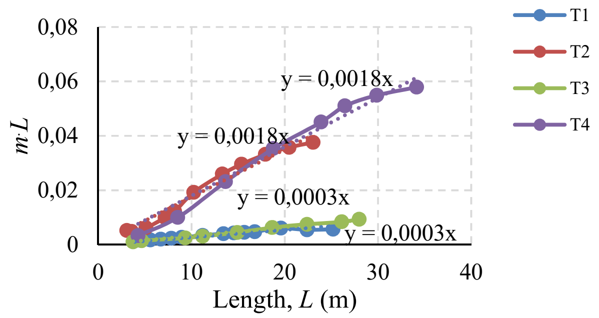

Fig. 7 Approximated values for m (specimens T1-T4)

TABLE IV
APPROXIMATED M VALUES

Element type	m
T1	0.0003
T2	0.0018
T3	0.0003
T4	0.0018
T5	0.0006
T6	0.0021
T7	0.0003
T8	0.0014

The results shown in Table IV and in Fig. 7 prove that the inclination of m is considerably lower in the case of girders having corrugation angle of 30°, than that of the girders having corrugation angle of 45°. Consequently, the corrugation angle has a major effect on the value of the C_{tr} modification factor. According to the previous studies, apart from the corrugation angle, the geometry of the corrugation and the flanges also may have influence on the critical moment. A ratio introduced by Johnson and Cafolla [5] is shown in (14) and explained in Fig. 8, which is an applicable and logical way to consider the effect of the corrugation profile [13].

$$R = \frac{A_1}{A_2} \,, \tag{14}$$

where $A_1 = (a_1 + a_4) \cdot a_3$, $A_2 = (a_1 + 2 \cdot a_4) \cdot b_f$.

Fig. 9 shows the calculated values of m on the vertical axis and $R \cdot tg(\alpha)$ on the horizontal axis.

The values of $R \cdot tg(\alpha)$ seem to be in correlation with the values of m. The trendline has a gradient of 0.0096, which may be approximated as 0.01. It means that the value of C_{tr}

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can be approximated by (15) considering the effect of the corrugation profile as well.

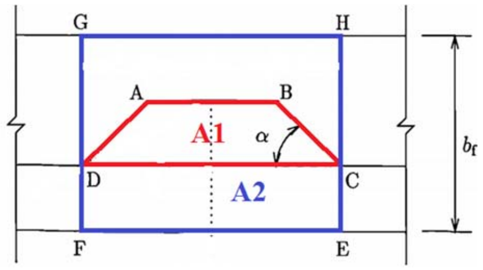

Fig. 8 Description of *R*

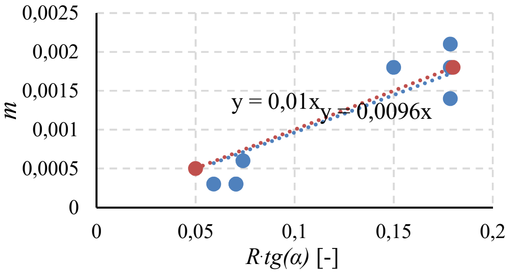

Fig. 9 Correlation of m values with *R*tg(α)*

$$C_{tr} = 1.0 + \sqrt{0.01 \cdot R \cdot tg(\alpha) \cdot L}\,. \tag{15}$$

Fig. 10 shows the results using the modification factor introduced in (15). On the vertical axis, the values of the standard based critical moment ($M_{cr,EC}$) can be seen divided by the numerical critical moment of the TWGs. The length of the girder is presented on the horizontal axis. The numerical critical moments of flat web girders are denoted by $M_{cr,num,flat}$. The $M_{cr,EC}$ represents the critical moment calculated according to the recommendations of the EN1993-1-1 for flat web girders. The $M_{cr,cal,num}$ is calculated according to (16).

$$M_{cr,cal,num} = C_{tr} \cdot M_{cr,num,flat}\,. \tag{16}$$

The values of $M_{cr,cal,num}$ are shown in Fig. 10 having ±7% deviation from the value of $M_{cr,num,tr}$, so the modification factor, which is introduced in (15) could be an appropriate and efficient way to consider the LTB critical moment of the TWGs. This modification factor takes into account the difference between the structural behavior of the TWGs and the flat web girders.

Fig. 11 presents the comparison of the FEM based critical bending moment and the standard based (blue points) and modified FEM based critical bending moment (red points). It can be observed the standard based critical bending moment formula (developed for flat web girders) provides conservative solutions with a large scatter. However, by using the proposed modification factor, the critical bending moment of the TWGs can be approximated with large accuracy.

D.Results of the GMNI analysis

Fig. 12 shows the overall results of the geometrical and material nonlinear imperfect analysis (GMNIA). The buckling curves *c* and *d* of the EN1993-1-1 [11] are also presented on the diagram for comparison purposes. The horizontal axis represents the relative slenderness ($\tilde{\lambda}_{LT}$) related to LTB, and the vertical axis represents the reduction factor (χ_{LT}), which is calculated by the ratio of the computed LTB resistance of the studied members and the bending moment resistance of the analyzed cross-section. The blue points represent the results regarding the standard based critical bending moment formula in the relative slenderness calculation, while the red points represent the results regarding the FEM based critical bending moment in the relative slenderness calculation. It can be observed that the standard based procedure – developed for flat web girders – provide too conservative results with large deviations in the LTB resistance calculation. The application of the FEM based (or by the C_{tr} modified) critical bending moment values, however, provides results with smaller deviations (red points) and results near to the column buckling curve c.

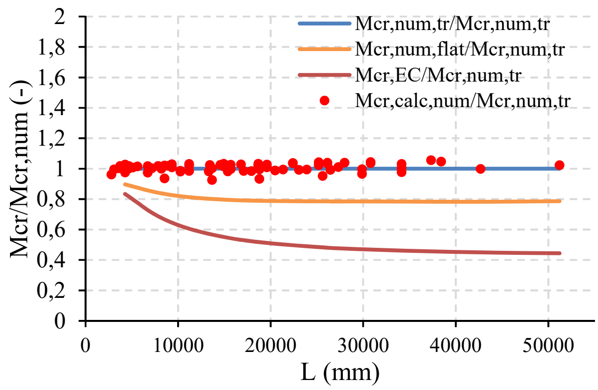

Fig. 10 Results from calculating with *Ctr*

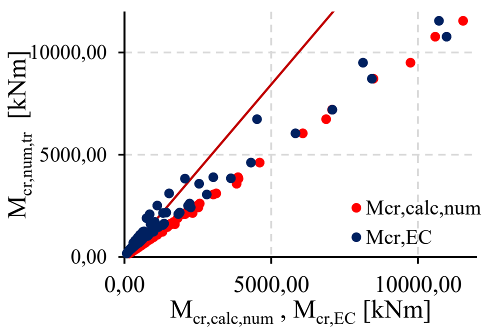

Fig. 11 Effect of the modification factor *Ctr*

The results prove that all computed simulation results (red points) are located above the flexural buckling curve *d* and 97% of the points are located above and very close to the buckling curve *c* as well. It indicates that the buckling curve *c* might be applicable for TWGs if the proposed modification factor is applied.

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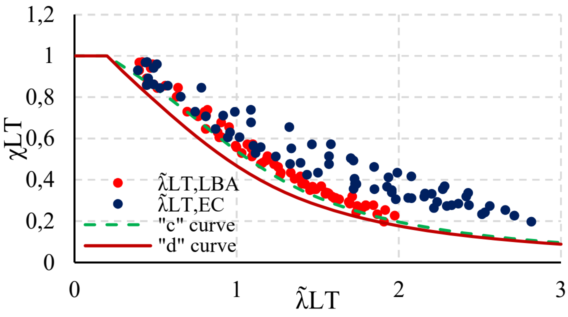

Fig. 12 Results and the LTB curves

V. Conclusion

Numerous researches have shown that TWGs can have higher resistance against LTB than girders with flat webs. However, the current codes do not have any recommendations for calculating the LTB resistance of TWGs.

After studying the previous research papers concerning this topic, a numerical model is built and validated based on experimental test results coupled with imperfection sensitivity analysis.

In frame of the executed research program, linear buckling analysis and geometrical and material nonlinear imperfect analysis (GMNIA) are carried out to determine the critical bending moment and the LTB resistance of the analyzed girders.

Through the results of the linear buckling analysis, a parametric study is conducted to investigate the physical background of the difference between the TWGs and the conventional girders with flat web. A modification factor is developed and introduced considering the difference in the LTB behavior and in the calculation of the LTB critical moment of the TWGs.

According to the results of the GMNIA simulations, it can be observed that the LTB buckling resistance of TWGs is more favorable in the analyzed parameter domain, than those of with flat web. It is observed that all the numerical results are above the flexural buckling curve *d* of the EN1993-1-1 [11] (so the curves tend to be on the safe side). In addition, buckling curve *c* might be also applicable after performing a safety assessment procedure to prove its applicability.

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