World Academy of Science, Engineering and Technology
International Journal of Physical and Mathematical Sciences
Vol:11, No:7, 2017

Nonlinear Propagation of Acoustic Soliton Waves in Dense Quantum Electron-Positron Magnetoplasma

A. Abdikian

Open Science Index, Physical and Mathematical Sciences Vol:11, No:7, 2017 publications.waset.org/10007503/pdf

***Abstract*—**Propagation of nonlinear acoustic wave in dense electron-positron (e-p) plasmas in the presence of an external magnetic field and stationary ions (to neutralize the plasma background) is studied. By means of the quantum hydrodynamics model and applying the reductive perturbation method, the Zakharov-Kuznetsov equation is derived. Using the bifurcation theory of planar dynamical systems, the compressive structure of electrostatic solitary wave and periodic travelling waves is found. The numerical results show how the ion density ratio, the ion cyclotron frequency, and the direction cosines of the wave vector affect the nonlinear electrostatic travelling waves. The obtained results may be useful to better understand the obliquely nonlinear electrostatic travelling wave of small amplitude localized structures in dense magnetized quantum e-p plasmas and may be applicable to study the particle and energy transport mechanism in compact stars such as the interior of massive white dwarfs etc.

***Keywords*—**Bifurcation theory, magnetized electron-positron plasma, phase portrait, the Zakharov-Kuznetsov equation.

I. Introduction

SINCE the past decade, the study of linear and the nonlinear electrostatic and electromagnetic waves in quantum plasmas using the quantum hydrodynamic (QHD) [1], [2] has highly come to the attention of researchers because they have many uses in several physical systems including ordinary metals, semiconductors, super dense astrophysical environments (e.g. neutron stars, white dwarfs, etc.), nano-devices, and laser-plasma experiments [1]-[6]. In these systems which have high densities, plasma acts as a degenerate fluid, and quantum mechanical effects (i.e. when the average inter-particle distance is equal to or smaller than the de Broglie thermal wavelength of the charged particles) play an important role in the plasma dynamics [7]. One can generalize the QHD model by adding the quantum statistical pressure term (the Fermi-Dirac distribution), the quantum diffraction term (the Bohm potential) and the exchange-correlation effects [8]-[11] to the fluid model. By studying the nonlinear wave phenomenon in these systems, it is demonstrated that the inclusion of quantum effects is important to study a nonlinear wave phenomenon in a quantum plasma [1], [2]. The QHD was used to investigate the quantum ion acoustic waves, and a deformed Korteweg-de Vries (KdV) equation was derived by Haas et al. [12]. Although the nonlinear effects can result in a shock formation due to the large amplitude of oscillations, in the presence of dispersion effects in the system they balance each other, and soliton structure will be emerged. In classical plasmas, the KdV equation is well-known for small but finite amplitudes for ion acoustic wave [13], [14]. In quantum plasma, several authors have studied linear and nonlinear low-frequency waves such as ion acoustic waves, drift waves, and so on and so forth [12], [15]-[17].

It has been proved that by adding positrons to usual plasmas (including linear and nonlinear electrostatic and electromagnetic waves), their collective behavior has significantly changed [18]-[21]. The existence and significant role of electron-positron (e-p) plasmas (which consists of identical mass but opposite charged particles) in the early universe [22] is incontrovertible [21], [23]. Although the existing of the e-p plasma in most astrophysical environments can be considered in the relativistic regime [24], [25] (and the references therein), the collective behavior of e-p plasma in the nonrelativistic regimes is also significant for realization of some aspects of astrophysical plasma [23], [26]. Verheest et al. studied large amplitude solitary electromagnetic waves in electron-positron plasmas via a reductive perturbation analysis and obtained a modified Korteweg-de Vries (mKdV) equation [27]. Using a two-fluid plasma model, Kourakis et al. [28] studied the nonlinear propagation of electrostatic wave packets parallel to the external magnetic field in pair plasmas. With this approach, Esfandyari-Kalejahi et al. considered the nonlinear propagation of amplitude-modulated electrostatic wave-packets in e-p-i plasma [29]. Esfandyari-Kalejahi et al. [30] studied the nonlinear amplitude modulation of electrostatic waves which propagate in unmagnetized collisionless pair plasma.

Furthermore, the solitary wave structures in magnetized plasma have been studied by many researchers, and they derived Zakharov-Kuznetsov (ZK) equation in different mediums. For instance, Kourakis et al. have studied the nonlinear propagation of electrostatic excitations in rotating magnetized doped pair-ion plasmas and they have derived the equation on the formation of multidimensional solitons [31]. The propagation of the shear Alfvén waves in a strongly magnetized e-p-i plasmas has been investigated by Yu et al. [32], and also the solitary waves in quantum e-p-i plasmas were investigated [15], [23], [33]. Mahmood et al., by employing the QHD, have studied ZK equation for nonlinear acoustic wave propagation in dense magnetized e-p plasmas in the presence of stationary ions and found that an increase in positron concentration decreases the wave amplitude [23]. Moreover, the dynamics of linear and nonlinear ionic-scale electrostatic excitations propagating in magnetized and unmagnetized relativistic quantum plasma have been studied,

A. Abdikian is with the Department of Physics, Malayer University, Malayer 65719-95863, Iran (e-mail: abdykian@malayeru.ac.ir).

Open Science Index, Physical and Mathematical Sciences Vol:11, No:7, 2017 publications.waset.org/10007503/pdf

and ZK and KdV equations have been derived, respectively [34]-[36]. On the other hand, in recent years, many researchers are interested to study the solitary waves by bifurcation theory of planar dynamical systems [37]-[40] (see references there in). The bifurcation theory [41] is a well-known powerful technique for studying the dynamical behavior for several models of plasmas. Samanta et al. [37], by applying the bifurcation theory and using the reductive perturbation method, have derived a Kadomtsev-Petviashili (KP) equation for dust ion acoustic waves in a magnetized dusty plasma with *q*-nonextensive velocity distributed electrons, and the existence of two solitary wave and periodic travelling wave solutions is proved. They obtained the parameters that affect the nature of solitary waves and periodic travelling waves.

Samanta et al. [42] have investigated ion acoustic waves in two component plasma with cold ions and kappa distributed electron in the presence of an external static magnetic field and derived the ZK equation. Very recently, El-Shamy [39] analyzed nonlinear ion-acoustic cnoidal waves in a dense relativistic degenerate magnetoplasma consisting of relativistic degenerate electrons and nondegenerate cold ions. He has analytically derived modified KdV equation and by means of the Sagdeev potential approach, numerically studied the various solutions of nonlinear ion-acoustic cnoidal and solitary waves. With that method, El-Shamy et al. [40] have studied the nonlinear propagation of electrostatic travelling wave structures in degenerate dense magnetoplasmas consisting of relativistic degenerate inertialess electrons and positrons, as well as nondegenerate inertial cold ions and obtained the ZK equation. The aim of this paper is to investigate the bifurcation behavior of acoustic traveling waves in the quantum magnetized e-p plasma. We obtain solitary and periodic wave solution of the ZK equation.

The paper is organized as follows. In Section II, the basic equations for a quantum e-p magnetoplasma have given, and using the reductive perturbation method, the ZK equation has derived. Section III is devoted to the equilibrium points which obtained by bifurcation theory. The possibility of the existence of solitary wave structures and periodic travelling wave solutions are discussed in Section IV. Section V reported the numerical analysis and results. Finally, the conclusion is presented in Section VI.

II. Mathematical Model and Derivation of the Nonlinear Equation

We consider a fully ionized three-dimensional collisionless plasma with stationary ions, the quantum e-p plasma in the presence of the external magnetic field is directed along the *x*-axis, i.e., $\mathbf{B}_0 = B_0\hat{\mathbf{x}}$. The wave phase velocity $v_{ph} = \omega/k$ is assumed to be in the range $v_{ph} << v_{Fe}, v_{Fp}$, where $v_{Fj} = (2\varepsilon_{Fj}/m_e)^{1/2}$ (here *j=e, p*) are the ion thermal speed and electron/positron Fermi speed, respectively, and $\varepsilon_{Fj} = \hbar^2(3\pi^2 n_{j0})^{2/3}/2m_e$ is the Fermi energy of the *j*th species. The dynamic equations for magnetized quantum electron and positron plasmas are governed by the continuity, the momentum-balance and the Poisson equations [23], [31], [33], [36]:

$$\frac{\partial n_j}{\partial t} + \vec{\nabla}\cdot\left(n_j\,\vec{\mathbf{v}}_j\right) = 0\,, \tag{1}$$

$$\frac{\partial \vec{\mathbf{v}}_j}{\partial t} + \left(\vec{\mathbf{v}}_j\cdot\nabla\right)\vec{\mathbf{v}}_j = \frac{q_j}{m_e}\left[-\vec{\nabla}\Phi + \frac{1}{c}\vec{\mathbf{v}}_j\times\vec{\mathbf{B}}_0\right] - \frac{\vec{\nabla}P_j}{m_e n_j} + \frac{\hbar^2}{2m_e}\vec{\nabla}\left(\frac{\nabla^2\sqrt{n_j}}{\sqrt{n_j}}\right) - \frac{\vec{\nabla}V_{xc}}{m_e}\,, \tag{2}$$

$$\nabla^2\Phi = -4\pi e\left(n_p - n_e + n_{i0}\right) \tag{3}$$

where Φ is the electrostatic potential and perturbed densities (n_j) and velocities ($\vec{\mathbf{v}}_j$) of the *j*th species, $q_e = -e$ ($q_p = +e$) is the electric charge for electron (positron), $\hbar$ is the Planck constant (*h*) divided by 2π, and *c* is the speed of light in vacuum. The third term in (2) is Bohm potential, which appears to be due to tunneling effects in quantum plasmas [43]. The Fermi pressure for electron and positron quantum fluids is defined as $p_j = (mv_{Fj}^2)/(5n_{j0}^{2/3})n_j^{5/3}$. In a degenerate gas, the Fermi temperature and density of the *j*th species are related as $k_B T_{Fj} = \hbar^2(3\pi^2 n_{j0})^{2/3}/2m$, and T_{Fj} is the particle temperature, k_B is the Boltzmann constant. In equilibrium state, we have $n_{p0} + n_{i0} = n_{e0}$, where n_{e0}, n_{p0}, and n_{i0} are the unperturbed (equilibrium) densities of electrons, positrons and ions, respectively. The last term in (2) is the electron exchange-correlation potential which is a function of electron density and is given by

$$V_{xc} = -0.985\frac{n^{1/3}e^2}{\varepsilon}\left[1 + \frac{0.034}{n^{1/3}a_B}\ln\left(1 + 18.37n^{1/3}a_B\right)\right] \tag{4}$$

which can be obtained via the adiabatic local-density approximation, where $a_B = \varepsilon\hbar^2/m_e e^2$ is the Bohr radius, and $\varepsilon = 4\pi\varepsilon_0$ is the effective dielectric permeability of material [11], [44]-[47]. Since $18.37n^{1/3}a_B << 1$, Taylor expanding up to second order, (4) turns to

$$V_{xc} \approx -1.6(n^{1/3}e^2)/\varepsilon + 5.65(\hbar^2/m_e)n^{2/3}, \tag{5}$$

Now, we assume that wave propagation is in two dimensions, i.e. $\nabla = (\partial_x, \partial_y, 0)$, and we use the following normalized parameters to normalize (1)-(3)

Open Science Index, Physical and Mathematical Sciences Vol:11, No:7, 2017 publications.waset.org/10007503/pdf

$$t \to t\,\omega_{pe},\ x \to \frac{\omega_{pe}}{\lambda_{Fe}}x, \Phi \to \frac{e\Phi}{2k_B T_{Fe}}, v_j \to v_j / v_{Fe},$$
$$n_j \to n_j / n_{j0}, \omega_{pj} = \sqrt{\frac{4\pi e^2 n_{j0}}{m_e}}, H_j = \frac{\hbar\,\omega_{pj}}{2k_B T_{Fj}}, \quad (6)$$
$$\delta = \frac{n_{i0}}{n_{e0}}, \omega_c = \frac{eB_0}{mc}, \sigma_T = \frac{T_{Fp}}{T_{Fe}}, \Omega_c = \frac{\omega_c}{\omega_{pe}},$$
$$\lambda_{Fe} = \sqrt{\frac{2k_B T_{Fe}}{4\pi e^2 n_{e0}}}, \gamma_{xcj} = \frac{1.6\,n_{j0}^{1/3}e^2}{2k_B T_{Fj}\varepsilon}, \lambda_{xcj} = \frac{5.65\hbar^2 n_{j0}^{2/3}}{2k_B T_{Fj} m_e}$$

It should be noted that $0 < \delta < 1$. The normalized equations for the electron quantum fluid in the component form can be described as

$$\frac{\partial n_e}{\partial t} + \frac{\partial}{\partial x}(n_e v_{ex}) + \frac{\partial}{\partial y}(n_e v_{ey}) = 0, \quad (7)$$

$$\frac{\partial v_{ex}}{\partial t} + \left(v_{ex}\frac{\partial}{\partial x} + v_{ey}\frac{\partial}{\partial y}\right)v_{ex} = \frac{\partial \Phi}{\partial x} - \frac{1}{5n_e}\frac{\partial n_e^{5/3}}{\partial x} + \frac{H_e^2}{2}\frac{\partial}{\partial x}\left(\frac{1}{\sqrt{n_e}}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\sqrt{n_e}\right) - \lambda_{xce}\frac{\partial n_e^{2/3}}{\partial x} + \gamma_{xce}\frac{\partial n_e^{1/3}}{\partial x} \quad (8)$$

$$\frac{\partial v_{ey}}{\partial t} + \left(v_{ex}\frac{\partial}{\partial x} + v_{ey}\frac{\partial}{\partial y}\right)v_{ey} = \frac{\partial \Phi}{\partial y} - \Omega_c v_{ez} - \frac{1}{5n_e}\frac{\partial n_e^{5/3}}{\partial y} + \frac{H_e^2}{2}\frac{\partial}{\partial y}\left(\frac{1}{\sqrt{n_e}}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\sqrt{n_e}\right) - \lambda_{xce}\frac{\partial n_e^{2/3}}{\partial y} + \gamma_{xcj}\frac{\partial n_e^{1/3}}{\partial y} \quad (9)$$

$$\frac{\partial v_{ez}}{\partial t} + \left(v_{ex}\frac{\partial}{\partial x} + v_{ey}\frac{\partial}{\partial y}\right)v_{ez} = \Omega_c v_{ey}, \quad (10)$$

and the normalized equations for the positron quantum fluid in the component form can be written as

$$\frac{\partial n_p}{\partial t} + \frac{\partial}{\partial x}(n_p v_{px}) + \frac{\partial}{\partial y}(n_p v_{py}) = 0, \quad (11)$$

$$\frac{\partial v_{px}}{\partial t} + \left(v_{px}\frac{\partial}{\partial x} + v_{py}\frac{\partial}{\partial y}\right)v_{px} = -\frac{\partial \Phi}{\partial x} - \frac{\sigma_T}{5n_p}\frac{\partial n_p^{5/3}}{\partial x} + \frac{H_p^2\sigma_T^2}{2(1-\delta)}\frac{\partial}{\partial x}\left(\frac{1}{\sqrt{n_p}}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\sqrt{n_p}\right) - \lambda_{xcp}\frac{\partial n_p^{2/3}}{\partial x} + \gamma_{xcp}\frac{\partial n_p^{1/3}}{\partial x} \quad (12)$$

$$\frac{\partial v_{py}}{\partial t} + \left(v_{px}\frac{\partial}{\partial x} + v_{py}\frac{\partial}{\partial y}\right)v_{py} = -\frac{\partial \Phi}{\partial y} + \Omega_c v_{pz} - \frac{\sigma_T}{5n_p}\frac{\partial n_p^{5/3}}{\partial y} + \frac{H_p^2\sigma_T^2}{2(1-\delta)}\frac{\partial}{\partial y}\left(\frac{1}{\sqrt{n_p}}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\sqrt{n_p}\right) - \lambda_{xcp}\frac{\partial n_p^{2/3}}{\partial y} + \gamma_{xcp}\frac{\partial n_p^{1/3}}{\partial y} \quad (13)$$

$$\frac{\partial v_{pz}}{\partial t} + \left(v_{px}\frac{\partial}{\partial x} + v_{py}\frac{\partial}{\partial y}\right)v_{pz} = -\Omega_c v_{py}, \quad (14)$$

The normalized form of the Poisson equation is

$$\nabla^2\Phi = n_e - n_p(1-\delta) - \delta \quad (15)$$

To obtain dynamic nonlinear equation for the electrostatic potential in magnetized quantum e-p-i plasmas, we employ the stretching of independent variables as [36], [48], [49]

$$X = \varepsilon^{1/2}(x - \lambda t), \quad y = \varepsilon^{1/2}y, \ T = \varepsilon^{3/2}t, \quad (16)$$

where ε is a small $(0 < \varepsilon << 1)$ expansion parameter characterizing the nonlinearity strength, and λ is the phase velocity of the wave normalized to be determined later. Now by expanding the perturbed quantities about their equilibrium values in powers of ε as follows and substituting them into (5)-(13),

$$\begin{aligned} n_j &= 1 + \varepsilon\, n_j^{(1)} + \varepsilon^2 n_j^{(2)} + \varepsilon^3 n_j^{(3)} \cdots, \\ v_{jx} &= \varepsilon\, v_{jx}^{(1)} + \varepsilon^2 v_{jx}^{(2)} + \varepsilon^3 v_{jx}^{(3)} \cdots, \\ v_{jy} &= \varepsilon^{3/2}\, v_{jy}^{(1)} + \varepsilon^2 v_{jy}^{(2)} + \varepsilon^{5/2}\, v_{jy}^{(1)} \cdots, \\ v_{jz} &= \varepsilon^{3/2}\, v_{jz}^{(1)} + \varepsilon^2 v_{jz}^{(2)} + \varepsilon^{5/2}\, v_{jz}^{(1)} \cdots, \\ \Phi &= \varepsilon\, \Phi^{(1)} + \varepsilon^2 \Phi^{(2)} + \varepsilon^3 \Phi^{(3)} \cdots, \end{aligned} \quad (17)$$

One can use the reductive perturbation method [50] to derive the perturbation expansions. By keeping terms of lowest order $(\varepsilon^{3/2})$ of the continuity and momentum equations of electrons and positrons, we get the following equations

$$n_e^{(1)} = -\frac{3}{\alpha_e + 3\lambda^2}\Phi^{(1)}, \quad (18)$$

$$v_{ex}^{(1)} = -\frac{3\lambda}{\alpha_e + 3\lambda^2}\Phi^{(1)}, \quad (19)$$

$$v_{ez}^{(1)} = \frac{1}{\Omega_c}\frac{3\lambda^2}{\alpha_e + 3\lambda^2}\frac{\partial \Phi^{(1)}}{\partial Y}, \quad (20)$$

and

$$n_p^{(1)} = \frac{3}{\alpha_p + 3\lambda^2}\Phi^{(1)}, \quad (21)$$

$$v_{px}^{(1)} = \frac{3\lambda}{\alpha_p + 3\lambda^2}\Phi^{(1)}, \quad (22)$$

$$v_{pz}^{(1)} = \frac{1}{\Omega_c}\frac{3\lambda^2}{\alpha_p + 3\lambda^2}\frac{\partial \Phi^{(1)}}{\partial Y}, \quad (23)$$

where $\alpha_e = \gamma_{xce} - 2\lambda_{xce} - 1\ (= v_{xce} - 1)$ and $\alpha_p = \gamma_{xcp} - 2\lambda_{xcp} - \sigma_T\ (= v_{xce} - \sigma_T)$. It is clear that $v_{ez}^{(1)}$

Open Science Index, Physical and Mathematical Sciences Vol:11, No:7, 2017 publications.waset.org/10007503/pdf

and $v_{pz}^{(1)}$ appears due to $\mathbf{E}\times\mathbf{B}_0$ drift in magnetized plasma. The linear phase speed of the acoustic wave in magnetized dense pair plasmas is obtained by using the lowest-order (ε) from the Poisson equation as $n_e^{(1)}-(1-\delta)n_p^{(1)}=0$ and substitute the expressions (18) and (23) in it, as follows

$$\lambda=\sqrt{\frac{\alpha_e(1-\delta)-\alpha_p}{3(2-\delta)}}, \tag{24}$$

Keeping the next higher order terms, i.e. $\varepsilon^{5/2}$, from the continuity and the momentum equation along the *x*-direction and the ε^2 order term of the momentum equation along the *z*-axis of electrons (positrons), we have

$$\partial_X v_{ex}^{(2)}-\lambda\partial_X n_e^{(2)}+\partial_Y v_{ey}^{(2)}+\partial_T n_e^{(1)}+\partial_X(n_e^{(1)}v_{ex}^{(1)})=0, \tag{25}$$

$$\begin{aligned}&-\lambda\partial_X v_{ex}^{(2)}-\partial_X\Phi^{(2)}-\alpha_e/3\,\partial_X n_e^{(2)}=\partial_T v_{ex}^{(1)}-v_{ex}^{(1)}\partial_X v_{ex}^{(1)}\\&-(v_{xce}+\alpha_e+\lambda_{xce})/9\,n_e^{(1)}\partial_X n_e^{(1)}+(H_e^2/4)\partial_X(\partial_Y^2+\partial_Y^2)n_e^{(1)}\end{aligned}, \tag{26}$$

$$v_{ey}^{(2)}=-(\lambda/\Omega_c)\partial_X v_{ez}^{(1)}, \tag{27}$$

$$\partial_X v_{px}^{(2)}-\lambda\partial_X n_p^{(2)}+\partial_Y v_{py}^{(2)}+\partial_T n_p^{(1)}+\partial_X(n_p^{(1)}v_{px}^{(1)})=0 \tag{28}$$

$$\begin{aligned}&-\lambda\partial_X v_{px}^{(2)}+\partial_X\Phi^{(2)}-\alpha_p/3\,\partial_X n_p^{(2)}=\\&\partial_T v_{px}^{(1)}-v_{px}^{(1)}\partial_X v_{px}^{(1)}-(v_{xcp}+\alpha_p+\lambda_{xcp})/9\,n_p^{(1)}\partial_X n_p^{(1)},\\&+(H_p^2\sigma_T^2/4(1-\delta))\partial_X(\partial_Y^2+\partial_Y^2)n_p^{(1)}\end{aligned} \tag{29}$$

$$v_{py}^{(2)}=\lambda/\Omega_c\,\partial_X v_{pz}^{(1)}, \tag{30}$$

However, the next higher order ε^2 term of the Poisson equation gives

$$n_e^{(2)}-(1-\delta)n_p^{(2)}=\partial_X^2\Phi^{(1)}+\partial_Y^2\Phi^{(1)}, \tag{31}$$

From (25)-(31), we obtain, after some simplification, the ZK equation for acoustic waves in magnetized e-p plasmas in the presence of stationary ions in terms of $\Phi^{(1)}$ as follows

$$\partial_T\Phi^{(1)}+A\Phi^{(1)}\partial_X\Phi^{(1)}+\partial_X\left(B\partial_X^2+C\,\partial_Y^2\right)\Phi^{(1)}=0, \tag{32}$$

where the nonlinear coefficient *A* and the dispersive coefficients *B* and *C* are defined as

$$A=\frac{3}{K}\left(\frac{(v_{xce}+\alpha_e+\gamma_{xce}+27\lambda^2)}{(\alpha_e+3\lambda^2)^3}-(1-\delta)\frac{(\alpha_p+\gamma_{xcp}+27\lambda^2-\sigma_T)}{(\alpha_p+3\lambda^2)^3}\right), \tag{33}$$

$$\begin{aligned}B=&\frac{1}{4K(\alpha_e+3\lambda^2)^2(\alpha_p+3\lambda^2)^2}\left(9H_e^2(\alpha_p+3\lambda^2)^2\right.\\&\left.-(\alpha_e+3\lambda^2)^2(4\alpha_p^2+24\alpha_p\lambda^2+36\lambda^4-9H_p^2\sigma_T^2)\right),\end{aligned} \tag{34}$$

$$\begin{aligned}C=&\frac{(\alpha_p+3\lambda^2)^2(\alpha_e+3\lambda^2)^2}{4K}\left\{\frac{9H_e^2}{(\alpha_e+3\lambda^2)^2}+\frac{9H_p^2\sigma_T^2}{(\alpha_p+3\lambda^2)^2},\right.\\&\left.-\frac{4}{\Omega_c^2}\left[\Omega_c^2+9\lambda^4\left(\frac{1}{(\alpha_e+3\lambda^2)^2}+\frac{1-\delta}{(\alpha_p+3\lambda^2)^2}\right)\right]\right\}\end{aligned}$$

where

$$K=-18\lambda\left(\frac{1}{(\alpha_e+3\lambda^2)^2}+\frac{1-\delta}{(\alpha_p+3\lambda^2)^2}\right). \tag{35}$$

III. Bifurcation of ZKB Equation

By introducing the transformation of the independent variables X, Y, and T into the one variable $\eta=\chi\left(l_x X+l_y Y-U\,T\right)$ where U is the normalized constant speed, and $l_x\,(l_y)$ is the direction cosine of the wave vector along the *x* (*y*) and $l_x^2+l_y^2=1$, we would be able to study the possibility of the existence of solitary wave solutions and periodic travelling wave solutions. By considering $\Psi(\eta)=\Phi^{(1)}(X,Y,T)$, integrating (32) with respect to η and neglecting the integration constant, one can derive the form of an ordinary differential equation as follows:

$$\frac{d^2\Psi}{d\eta^2}=a\,\Psi-b\,\Psi^2, \tag{36}$$

where $a=\dfrac{U}{\chi^2 l_x\left(l_x^2(B-C)+C\right)}$ and $b=\dfrac{Al_x/2}{\chi^2 l_x\left(l_x^2(B-C)+C\right)}$.
Now, one can rewrite (34) as the following dynamical system of travelling wave equations varied by plasma parameters

$$\begin{cases}\dfrac{d\Psi}{d\eta}=z\\[2mm]\dfrac{dz}{d\eta}=(a-b\Psi)\Psi\end{cases}, \tag{37}$$

The last equation defines a planar Hamiltonian system with the following Hamiltonian function

$$H(\Psi,z)=\frac{z^2}{2}-\left(\frac{a}{2}-\frac{b}{3}\Psi\right)\Psi^2=h_1, \tag{38}$$

It should note that the phase orbits determined by the vector fields of (35) define all traveling wave solutions of (32). By changing the plasma parameters, one can investigate the bifurcations of phase portrait of (35) in the (Ψ,z) phase plane. So, our studies have been restricted to the bounded traveling wave solutions of (32). According to the bifurcation theory, a homoclinic orbit of (35) corresponds to a solitary wave

Open Science Index, Physical and Mathematical Sciences Vol:11, No:7, 2017 publications.waset.org/10007503/pdf

solution of (32). A periodic orbit of (35) corresponds to a periodic traveling wave solution of (32) [41], [51], [52]. Using the bifurcation theory [41], [51], [52] of phase portraits of (35), there are two equilibrium points at $E_0(\Psi_0,0)$ and $E_1(\Psi_1,0)$, where $\Psi_0 = 0$ and $\Psi_1 = 2U/(l_x A)$. If we consider that $M(\Psi_i,0)$ is the coefficient matrix of the linearized system of (35) at an equilibrium point $E_i(\Psi_i,0)$, then we have

$$J = \det M(\Psi_i,0) = -a + 2b\Psi_i,$$

By the theory of planar dynamical systems [41], [51], [52], it is well known that the equilibrium point $E_i(\Psi_i,0)$ of the Hamiltonian system will be a saddle point, a center or Poincaré index if $J<0$, $J>0$ or $J=0$ (cusp point), respectively.

IV. Exact Explicit Travelling Wave Solution of ZK Equation

By applying the planar dynamical system (35) and Hamiltonian (36) with $h_1 = 0$, one can obtain that there are two types of solitary wave solution and periodic travelling wave solution of (32) for described magnetoplasma systems depending on different parameters.

When the condition $(1-\delta)\left(\frac{\alpha_e + 3\lambda^2}{\alpha_p + 3\lambda^2}\right)^3 > \frac{\alpha_e + \gamma_{xce} + \nu_{xce} + 27\lambda^2}{\alpha_p + \gamma_{xcp} + 27\lambda^2 - \sigma_T}$ is satisfied, (32) has the following compressive solitary wave solution [36], [40], [42]

$$\Psi(\eta) = \phi_m \sec h^2(\eta/W), \tag{39}$$

where $\phi_m = 3U/(l_x A)$ is the maximum amplitude and is $W = \sqrt{4\chi^2 l_x (B l_x^2 + C l_y^2)/U}$ width of solitary wave in a magnetic quantum plasma with degenerated electrons and positrons.

Otherwise, (32) has the periodic travelling wave solution in terms of Jacobian elliptic functions [38]-[40], [42]

$$\Psi(\eta) = \Psi_1 + \Psi_{sn} Sn^2(D\eta, m), \tag{40}$$

where $\Psi_{sn} = \Psi_0 - \Psi_1$, $D = \sqrt{\frac{A l_x (\Psi_0 - \Psi_2)}{12\chi^2 l_x \left(l_x^2 (B-C) + C\right)}}$ and $m^2 = \frac{\Psi_0 - \Psi_1}{\Psi_0 - \Psi_2}$. It important to notice that the elliptic parameters (m) and the periodic solution, (39), satisfy the conditions $0 < m < 1$ and $\Psi_0 > \Psi_1 > \Psi_2$, respectively.

V. Numerical Results and Discussion

In this section, we have studied numerically the effects of some parameters on the characteristics of solitary wave and periodic travelling solutions such as the ion to electron equilibrium density ratio δ, the electron cyclotron to electron plasma frequency ratio Ω_c, and the direction cosines of the wave vector along the *x* axis l_x. For this purpose, we choose some of the typical plasma parameters found in astrophysical environments in which they are important [15], [23], [33], n_{e0}=5.9×10^{28}cm^{-3}, n_{p0}=5.32×10^{28}cm^{-3}, n_{i0}=5.8×10^{27}cm^{-3} and B_0=10^{9}G. The Fermi temperatures of electrons and positrons at such densities are $T_{Fe} = 1.96\times10^8 K$ and $T_{Fp} = 1.69\times10^8 K$; the quantum parameters for electrons and positrons at these densities are $H_e = 0.1097$ and $H_p = 0.1138$; Fermi lengths are $\lambda_{Fe} = 1.367\times10^{-9} cm$ and $\lambda_{Fp} = 1.419\times10^{-9} cm$.

By helping of the systematic analysis, the phase portraits of the dynamical system of travelling wave, i.e. (35), are plotted in Figs. 1 and 2. They show that waves depend numerically on parameters δ and Ω_c.

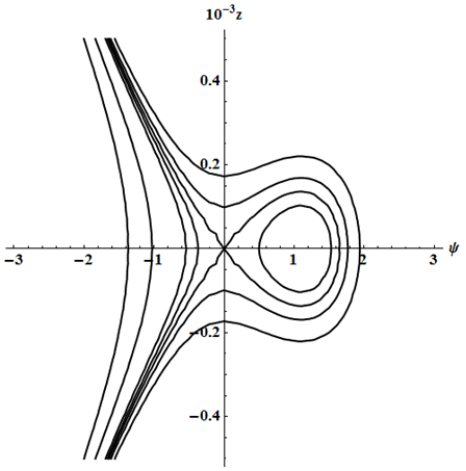


Fig. 1 Phase orbits with $\chi = 2$, $\delta = 0.2$, $l_x = 0.2$, $\gamma_{xce,p} = 0.4$, $\lambda_{xce,p} = 0.1$, $\Omega_c = 0.0003$ and $U = 2.17$

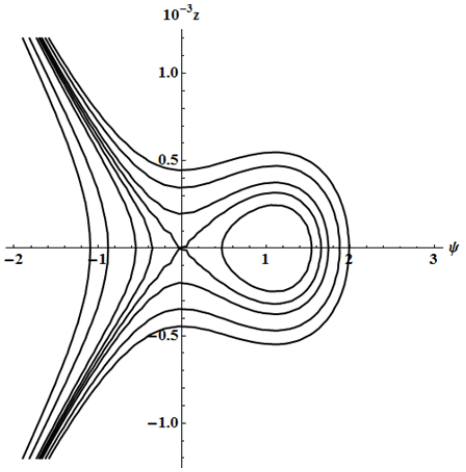


Fig. 2 Phase curves for $\chi = 2$, $\delta = 0.2$, $l_x = 0.2$, $\gamma_{xce,p} = 0.4$, $\lambda_{xce,p} = 0.1$, $\Omega_c = 0.007$ and $U = 2.17$

Fig. 1 illustrates phase portrait of (35) for some particular parameter values. For plotting these figures, by considering the following relationship $(1-\delta)\left(\frac{\alpha_e+3\lambda^2}{\alpha_p+3\lambda^2}\right)^3 > \frac{\alpha_e+\gamma_{xce}+\nu_{xce}+27\lambda^2}{\alpha_p+\gamma_{xcp}+27\lambda^2-\sigma_T}$, the A coefficient is always positive. Hence, they suggest that there exists a homoclinic orbit (at the equilibrium point $E_0(\Psi_0,0)$ which it is a saddle point) and a family of periodic orbits ($E_1(\Psi_1,0)$ which it is a center point).

Fig. 2 illustrates phase portrait of (35) for some particular parameter values. The three-dimensional profile of the amplitude and the width of the quantum e-p acoustic solitary versus the ion equilibrium density ratio δ is plotted in Fig. 3. For plotting, the obliqueness of the wave l_x=0.4, $\gamma_{xce,p}=0.4$, $\lambda_{xce,p}=0.1$, $U=1.4$ with the electron cyclotron ratio ($\Omega_c=0.004$) is chosen. It is found that the both wave amplitude and width of the soliton are increased with decreasing of δ. This behavior arises due to the fact that the electron pressure creates the restoring force of solitary wave, which enhances by increasing (decreasing) the value of ion (positron) density ratio. Hence, the increase of restoring force leads to an increase in the amplitude of solitary wave. The result shows that the ion (positrons) density ratio has an important effect in the solitary wave propagation.

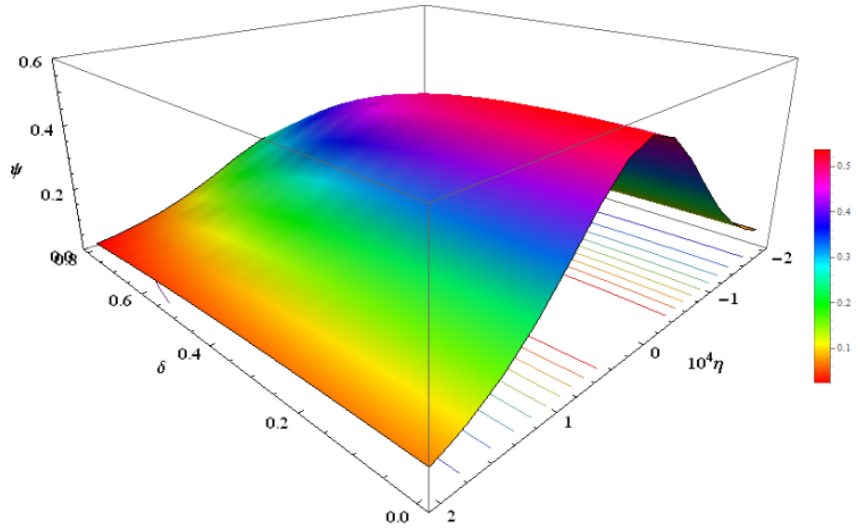


Fig. 3 The variations of solitary wave versus δ for $l_x=0.4$, $\gamma_{xce,p}=0.4$, $\lambda_{xce,p}=0.1$, $\Omega_c=0.004$ and $U=1.4$

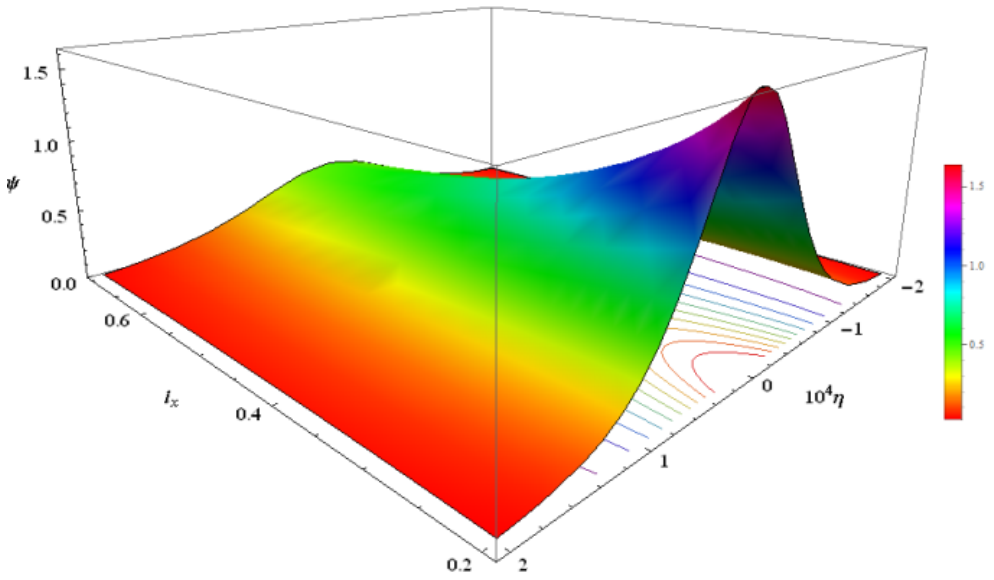


Fig. 4 The variations of solitary wave versus l_x with $\delta=0.2$, $\gamma_{xce,p}=0.4$, $\lambda_{xce,p}=0.1$, $\Omega_c=0.004$ and $U=2.17$

The effect of the direction cosines of the wave vector along the x axis l_x on the profile of solitary wave is investigated in

Open Science Index, Physical and Mathematical Sciences Vol:11, No:7, 2017 publications.waset.org/10007503/pdf

Fig. 4. For plotting, the ion equilibrium density ratio $\delta = 0.2$, $\gamma_{xce,p} = 0.4$, $\lambda_{xce,p} = 0.1$, $U = 2.17$ with the electron cyclotron ratio ($\Omega_c = 0.004$) are chosen. It is found that the both wave amplitude and width of the soliton are increased as the ion equilibrium density ratio δ is enhanced.

Fig. 5 shows the variation of the amplitude and the width of the quantum e-p acoustic solitary versus the electron cyclotron ratio Ω_c. As it can be seen, by increasing the value of the electron cyclotron frequency Ω_c, the width of solitary wave increased.

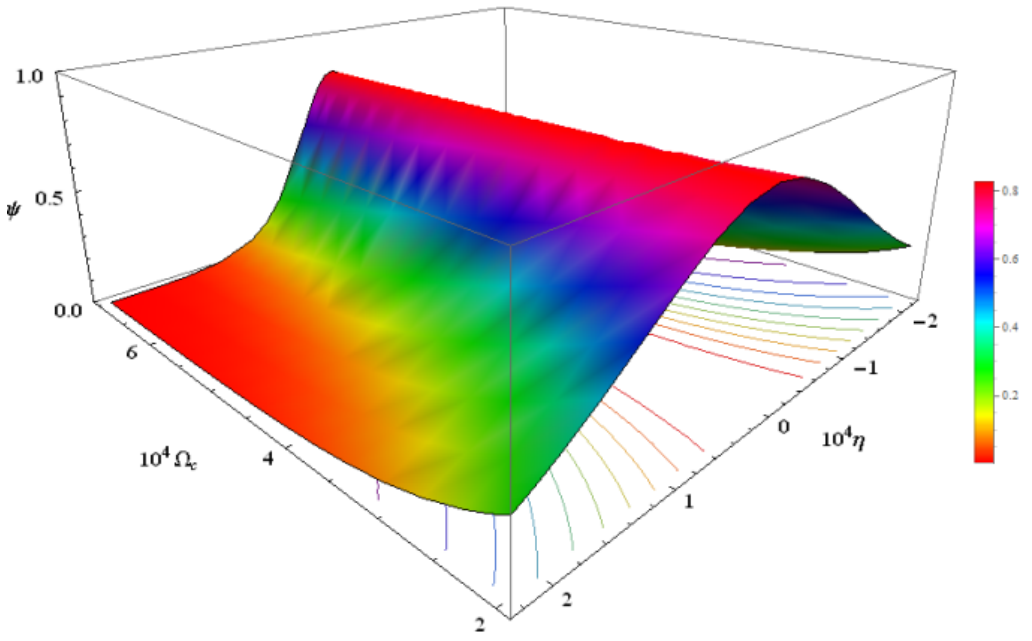


Fig. 5 The variations of solitary wave versus Ω_c with $\delta = 0.2$, $\gamma_{xce,p} = 0.4$, $\lambda_{xce,p} = 0.1$, $l_x = 0.4$ and $U = 2.17$.

The periodic travelling wave solutions of (32) are plotted versus of different parameters in Figs. 6-8.

The periodic travelling wave solutions (Ψ) with respect to the space coordinate (η) are plotted for two different values of the obliqueness of the wave, i.e. $l_x = 0.4$ and $l_x = 0.5$ in Fig. 6. For plotting, the following parameters $\chi = 2$, $\delta = 0.2$, $\gamma_{xce,p} = 0.4$, $\lambda_{xce,p} = 0.1$, $U = 1.4$ with the electron cyclotron ratio ($\Omega_c = 0.004$) are chosen. It is observed that the increasing l_x causes to the both amplitude and the width of solitary wave decrease.

The effects of the ion equilibrium density ratio δ on the periodic travelling wave solution are observed in Fig. 7. The value of parameters for plotting $\chi = 2$, $l_x = 0.4$, $\gamma_{xce,p} = 0.4$, $\lambda_{xce,p} = 0.1$, $U = 0.4$ with the electron cyclotron ratio ($\Omega_c = 0.004$) are chosen. Comparing the two cases, it is easily found that the amplitude and the width of the periodic wave grow up due to the increase of the ion density ratio δ. Physically, the driving force produced by ion inertia which increases by the reduction in the positron of the plasma system, leads to increase the amplitudes of the periodic traveling waves.

Fig. 7 exhibits the impact of the electron cyclotron frequency Ω_c on the periodic traveling waves. It is seen that the amplitude and width of the periodic travelling waves are enhancing with the increasing of Ω_c.

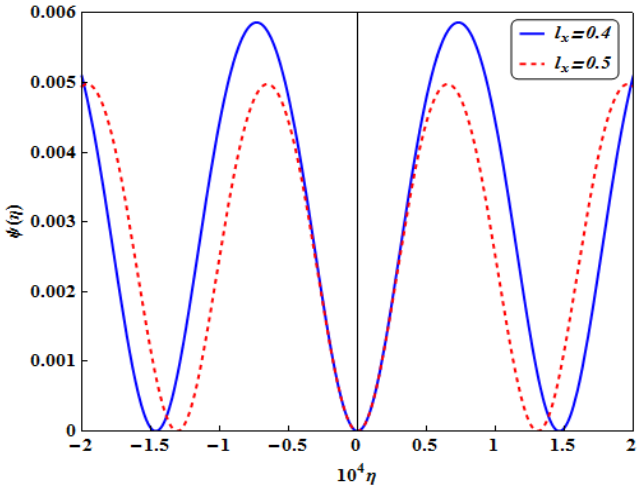


Fig. 6 Variation of periodic travelling wave Ψ versus η for $\chi = 2$, $\delta = 0.2$, $\gamma_{xce,p} = 0.4$, $\lambda_{xce,p} = 0.1$, $U = 0.4$ and $\Omega_c = 0.004$

Open Science Index, Physical and Mathematical Sciences Vol:11, No:7, 2017 publications.waset.org/10007503/pdf

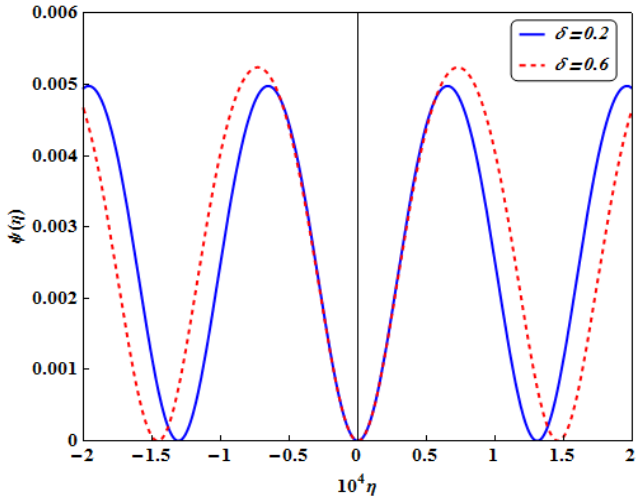


Fig. 7 Variation of periodic travelling wave Ψ versus η for $\chi = 2$, $l_x = 0.4$, $\gamma_{xce,p} = 0.4$, $\lambda_{xce,p} = 0.1$, $U = 0.4$ and $\Omega_c = 0.004$

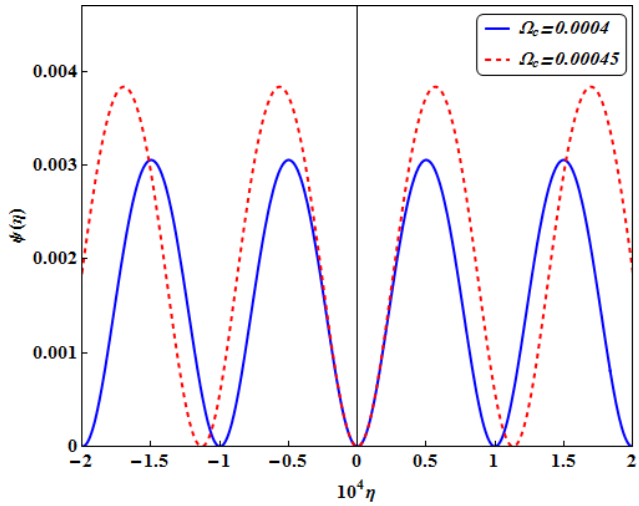


Fig. 8 Variation of periodic travelling wave Ψ versus η for $\chi = 2$, $\delta = 0.2$, $l_x = 0.2$, $\gamma_{xce,p} = 0.4$, $\lambda_{xce,p} = 0.1$ and $U = 0.4$

VI. Conclusion

We have investigated the obliquely nonlinear electrostatic wave structures in a dense quantum e-p plasma in the presence of the external magnetic field (is directed along the *x*-axis, i.e., $\mathbf{B}_0 = B_0\hat{\mathbf{x}}$) with stationary ions. By using the standard reductive perturbation technique and the hydrodynamics model for the dynamic of the fluid e-p, the Zakharov-Kuznetsov (ZK) equation, which represents the dynamics of small as well as finite amplitudes of quantum e-p acoustic solitary wave, is founded. The bifurcation theory of planar dynamical systems was used to illustrate the compressive structure of electrostatic solitary wave and periodic travelling waves may exist in such plasmas. By the numerical results, the dependence of the nonlinear electrostatic travelling waves on the ion density ratio, the ion cyclotron frequency, and the direction cosines of the wave vector effect on the nonlinear electrostatic travelling waves have studied. The results may be important to study the obliquely nonlinear electrostatic travelling wave in dense magnetized quantum e-p plasmas which may exist in compact stars such as the massive white dwarfs, etc.

References

[1] B. Eliasson and P. K. Shukla, "Nonlinear Aspects of Quantum Plasma Physics: Nanoplasmonics and Nanostructures in Dense Plasmas," *Plasma and Fusion Research,* vol. 4, p. 032, 2009.
[2] P. K. Shukla and B. Eliasson, "Nonlinear aspects of quantum plasma physics," *Physics-Uspekhi,* vol. 53, p. 51, 2010.
[3] Y. D. Jung, "Quantum-mechanical effects on electron–electron scattering in dense high-temperature plasmas," *Phys. Plasmas,* vol. 8, p. 3842, 2001.
[4] G. Manfredi and F. Haas, "Self-consistent fluid model for a quantum electron gas," *Physical Review B,* vol. 64, p. 075316, 2001.
[5] W. Masood and B. Eliasson, "Electrostatic solitary waves in a quantum plasma with relativistically degenerate electrons," *Physics of Plasmas,* vol. 18, p. 034503, 2011.
[6] A. Abdikian and M. Bagheri, "Electrostatic waves in carbon nanotubes with an axial magnetic field," *Phys. Plasmas,* vol. 20, p. 102103, 2013.
[7] G. Manfredi, "How to model quantum plasmas," *Fields Inst. Commun.,* vol. 46, p. 263, 2005.
[8] W. Moslem, I. Zeba, and P. Shukla, "Solitary acoustic pulses in quantum semiconductor plasmas," *Applied Physics Letters,* vol. 101, p. 032106, 2012.
[9] Y. T. Ma, S. H. Mao, and J. K. Xue, "Waves in a bounded quantum plasma with electron exchange-correlation effects," *Physics of Plasmas,* vol. 18, p. 102108, 2011.
[10] A. Abdikian, "The influence of external transverse magnetic field in propagation of electrostatic oscillations in single-walled carbon nanotube," *The European Physical Journal D,* vol. 70, p. 218, 2016.
[11] A. Abdikian and Z. Ehsan, "Propagation of electromagnetic waves in quantum plasmas magnetized slab," *Physics Letters A,* p. in press, 2017.
[12] F. Haas, L. G. Garcia, J. Goedert, and G. Manfredi, "Quantum ion-acoustic waves," *Phys. Plasmas,* vol. 10, p. 3858, 2003.
[13] R. Sagdeev, "Cooperative phenomena and shock waves in collisionless plasmas," *Reviews of plasma physics,* vol. 4, p. 23, 1966.
[14] F. Chen, *Introduction to plasma physics and controlled fusion*. New York: Plenum Press, 1984.
[15] S. Ali, W. Moslem, P. Shukla, and R. Schlickeiser, "Linear and nonlinear ion-acoustic waves in an unmagnetized electron-positron-ion quantum plasma," *Physics of Plasmas,* vol. 14, p. 082307, 2007.
[16] A. Misra, P. Shukla, and C. Bhowmik, "Electron-acoustic solitary waves in dense quantum electron-ion plasmas," *Physics of Plasmas,* vol. 14, p. 082309, 2007.
[17] S. Mahmood and Q. Haque, "Nonlinear electrostatic waves in inhomogeneous dense dusty magnetoplasmas," *Physics Letters A,* vol. 374, p. 872, 2010.
[18] P. Shukla, B. Dasgupta, and P. Sakanaka, "Nonlinear magnetohydrodynamics of electron–positron plasmas," *Physics Letters A,* vol. 269, p. 144, 2000.
[19] S. Mahmood, A. Mushtaq, and H. Saleem, "Ion acoustic solitary wave in homogeneous magnetized electron-positron-ion plasmas," *New Journal of Physics,* vol. 5, p. 28, 2003.
[20] P. Shukla, B. Eliasson, and L. Stenflo, "Electromagnetic solitary pulses in a magnetized electron-positron plasma," *Physical Review E,* vol. 84, p. 037401, 2011.
[21] M. Momeni, "Linear and nonlinear electromagnetic waves in a magnetized quantum electron-positron plasma," *International Journal of Modern Physics C,* vol. 26, p. 1550058, 2015.
[22] S. I. Popel, S. V. Vladimirov, and P. K. Shukla, "Ion-acoustic solitons in electron-positron-ion plasmas," *Physics of Plasmas,* vol. 2, p. 716, 1995.
[23] S. Mahmood, N. Akhtar, and H. Ur-Rehman, "Acoustic solitons in magnetized quantum electron-positron plasmas," *Physica Scripta,* vol. 83, p. 035505, 2011.
[24] V. I. Berezhiani, V. Skarka, and S. Mahajan, "Relativistic solitary wave in an electron-positron plasma," *Physical Review E,* vol. 48, p. R3252, 11/01/ 1993.
[25] N. Iwamoto, "Electron-positron pair creation in relativistic shocks: Pair plasma in thermodynamic equilibrium," *Physical Review A,* vol. 39, p.

Open Science Index, Physical and Mathematical Sciences Vol:11, No:7, 2017 publications.waset.org/10007503/pdf

4076, 1989.
[26] G. Zank and R. Greaves, "Linear and nonlinear modes in nonrelativistic electron-positron plasmas," *Physical Review E,* vol. 51, p. 6079, 1995.
[27] F. Verheest and T. Cattaert, "Large amplitude solitary electromagnetic waves in electron-positron plasmas," *Physics of Plasmas,* vol. 11, p. 3078, 2004.
[28] I. Kourakis, A. Esfandyari Kalejahi, M. Mehdipoor, and P. K. Shukla, "Modulated electrostatic modes in pair plasmas: Modulational stability profile and envelope excitations," *Physics of Plasmas,* vol. 13, p. 052117, 2006.
[29] A. Esfandyari Kalejahi, I. Kourakis, M. Mehdipoor, and P. Shukla, "Electrostatic mode envelope excitations in e-p-i plasmas-application in warm pair ion plasmas with a small fraction of stationary ions," *Journal of Physics A: Mathematical and General,* vol. 39, p. 13817, 2006.
[30] A. Esfandyari Kalejahi, I. Kourakis, and P. Shukla, "Oblique modulation of electrostatic modes and envelope excitations in pair-ion and electron-positron plasmas," *Physics of Plasmas,* vol. 13, p. 122310, 2006.
[31] I. Kourakis, W. M. Moslem, U. M. Abdelsalam, R. Sabry, and P. K. Shukla, "Nonlinear dynamics of rotating multi-component pair plasmas and epi plasmas," *Plasma and Fusion Research,* vol. 4, p. 018, 2009.
[32] M. Yu, P. K. Shukla, and L. Stenflo, "Alfven vortices in a strongly magnetized electron-position plasma," *The Astrophysical Journal,* vol. 309, p. L63, 1986.
[33] Q. Haque, S. Mahmood, and A. Mushtaq, "Nonlinear electrostatic drift waves in dense electron-positron-ion plasmas," *Physics of Plasmas,* vol. 15, p. 082315, 2008.
[34] E. Behery, F. Haas, and I. Kourakis, "Weakly nonlinear ion-acoustic excitations in a relativistic model for dense quantum plasma," *Physical Review E,* vol. 93, p. 023206, 2016.
[35] M. A. Hossen and A. A. Mamun, "Nonlinear positron-acoustic waves in fully relativistic degenerate plasmas," *Physica Scripta,* vol. 91, p. 035201, 2016.
[36] A. Abdikian and S. Mahmood, "Acoustic solitons in a magnetized quantum electron-positron-ion plasma with relativistic degenerate electrons and positrons pressure," *Physics of Plasmas,* vol. 23, p. 122303, 2016.
[37] U. K. Samanta, A. Saha, and P. Chatterjee, "Bifurcations of dust ion acoustic travelling waves in a magnetized dusty plasma with a q-nonextensive electron velocity distribution," *Physics of Plasmas,* vol. 20, p. 022111, 2013.
[38] T. Kaladze and S. Mahmood, "Ion-acoustic cnoidal waves in plasmas with warm ions and kappa distributed electrons and positrons," *Physics of Plasmas,* vol. 21, p. 032306, 2014.
[39] E. El-Shamy, "Nonlinear ion-acoustic cnoidal waves in a dense relativistic degenerate magnetoplasma," *Physical Review E,* vol. 91, p. 033105, 2015.
[40] E. El-Shamy, R. Al-Chouikh, A. El-Depsy, and N. Al-Wadie, "Nonlinear propagation of electrostatic travelling waves in degenerate dense magnetoplasmas," *Physics of Plasmas,* vol. 23, p. 122122, 2016.
[41] S. N. Chow and J. K. Hale, *Methods of bifurcation theory* vol. 251. New York: Springer-Verlag, 1982.
[42] U. K. Samanta, A. Saha, and P. Chatterjee, "Bifurcations of nonlinear ion acoustic travelling waves in the frame of a Zakharov-Kuznetsov equation in magnetized plasma with a kappa distributed electron," *Physics of Plasmas,* vol. 20, p. 052111, 2013.
[43] F. Haas and S. Mahmood, "Linear and nonlinear ion-acoustic waves in nonrelativistic quantum plasmas with arbitrary degeneracy," *Physical Review E,* vol. 92, p. 053112, 11/25/ 2015.
[44] N. Crouseilles, P. A. Hervieux, and G. Manfredi, "Quantum hydrodynamic model for the nonlinear electron dynamics in thin metal films," *Physical Review B,* vol. 78, p. 155412, 10/09/ 2008.
[45] R. Maroof, A. Mushtaq, and A. Qamar, "Quantum dust magnetosonic waves with spin and exchange correlation effects," *Physics of Plasmas,* vol. 23, p. 013704, 2016.
[46] M. Shahmansouri, "The exchange-correlation effects on surface plasmon oscillations in semi-bounded quantum plasma," *Physics of Plasmas* vol. 22, p. 092106, 2015.
[47] K. Mebrouk, L. A. Gougam, and M. Tribeche, "Implication of the Electron Exchange-Correlation on Fully Nonlinear Quantum Dust Ion-Acoustic Solitons," *Communications in Theoretical Physics,* vol. 65, p. 73, 2016.
[48] N. A. El-Bedwehy and W. M. Moslem, "Zakharov-Kuznetsov-Burgers equation in superthermal electron-positron-ion plasma," *Astrophysics and Space Science,* vol. 335, p. 435, 2011.
[49] T. S. Gill, A. S. Bains, and N. S. Saini, "Ion acoustic soliton in weakly relativistic magnetized electron-positron-ion plasma," *Canadian Journal of Physics,* vol. 87, p. 861, 2009.
[50] R. C. Davidson and J. E. Scherer, "Methods in Nonlinear Plasma Theory," *IEEE Transactions on Plasma Science,* vol. 1, p. 58, 1973.
[51] H. Poincaré, "Sur l'équilibre d'une masse fluide animée d'un mouvement de rotation," *Acta mathematica,* vol. 7, p. 259, 1885.
[52] J. Guckenheimer and P. J. Holmes, *Nonlinear oscillations, dynamical systems, and bifurcations of vector fields* vol. 42. New York: Springer-Verlag, 1983.

Open Science Index, Physical and Mathematical Sciences Vol:11, No:7, 2017 publications.waset.org/10007503/pdf