

# Simulation and Validation of Spur Gear Heated by Induction using 3d Model

A. Chebak, N. Barka, A. Menou, J. Brousseau and D. S. Ramdenee

**Abstract**—This paper presents the study of hardness profile of spur gear heated by induction heating process in function of the machine parameters, such as the power (kW), the heating time (s) and the generator frequency (kHz). The global work is realized by 3D finite-element simulation applied to the process by coupling and resolving the electromagnetic field and the heat transfer problems, and it was performed in three distinguished steps. First, a Comsol 3D model was built using an adequate formulation and taking into account the material properties and the machine parameters. Second, the convergence study was conducted to optimize the mesh. Then, the surface temperatures and the case depths were deeply analyzed in function of the initial current density and the heating time in medium frequency (MF) and high frequency (HF) heating modes and the edge effect were studied. Finally, the simulations results are validated using experimental tests.

**Keywords**—induction heating / simulation / experimental validation / 3D model / hardness profile

## I. INTRODUCTION

THE complex nature of coupling the physical fields that compose the induction heating process compels industrials to develop trial and error procedures to obtain the desired case depth that is generally specified by the design criterion. The analytical models, that could predict hardness profiles, would facilitate process development and reduce largely the lead-time. However, the accuracy of these models is limited by approximations taken into account that not perfectly reproduce the non-linear behavior of the material properties. Moreover, these models are generally reserved for some limited applications and they cannot be used to understand the sensitivity of case depth versus machine parameters or material properties.

Consequently, it is a difficult task to reuse these results again in other applications [1-2]. So, a finite element (FE) model is necessary to perform an efficient study of induction heating process with good accuracy. On other hand, the parameters to be controlled on the induction machine are the

input power ( $P_M$ ) (kW), the heating time (s) and the generator frequency (kHz); while the simulation parameters are the initial current density ( $J_0$ ) ( $A/m^2$ ), the total computation time (s) and the frequency (kHz). It is then necessary to find a correlation between  $J_0$  and  $P_M$ . Moreover, the 2D FE model developed by some researchers could help to understand the induction process but it is not fully representative of the actual behavior of the process since it does not allow the evaluation of edge effect and the real power ratio between the machine power and the power consumed by the part since the last power is not expressed with precision [3, 4]. This model can be efficient in the case of long gear but becomes less relevant if thin parts are considered.

A 3D FE model coupling the electromagnetic field analysis and the thermal transfer problem is a good solution to investigate the edge effect and determinate the power ratio with more accuracy. This model allows understanding and confirming the experimental tests applied to this kind of gear. In this case, the current in the coil ( $J_0$ ) is imposed to reach maximal temperature value (simulation criterion) used to find the correlation between the initial current density and the power consumed by the gear during heating, called simulation power ( $P_S$ ). This power is evaluated by mathematical integration of the heat source created by induced currents. This power is useful because it is used to determinate the power ratio between  $P_S$  and  $P_M$ , that is very relevant to calibrate the developed models. In fact, the both powers are compared basing on the same hardness profiles obtained by simulation and experimental test by keeping -the same heating time and frequency [5].

Literature has demonstrated that some attempts of development of 3D model allow analyzing the hardness profile in function of the material properties or in terms of machine parameters [6]. However, these studies have considered the material properties dependence at equilibrium thermodynamics. The work done so far never refer to non-equilibrium conditions and consider that the behavior in equilibrium remains valid even under conditions where the heating and cooling is fast. Moreover, the actual behavior of the measured material properties is affected by the accuracy of measuring instruments and the data processing quality. In addition, heating for the majority of applications are made at average speeds of heating and the effect of heating rate has never been considered. Also, the effect of machine parameters on the hardness profile has never before been thoroughly covered and documented. Furthermore, the mesh used during these studies was often very coarse and cannot give a good accuracy of computation [7-8].

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The main objective of the current work is to study the hardness profile with machine parameters using 3D finite-element model. Indeed, this paper proposes some simulation efforts and deeply discusses the induction process based on temperature distribution into the gear. The surface temperature and the case depth are represented in function of the machine parameters. The convergence study is conducted with an aim to optimize the mesh and the assumptions used in modeling and the operating conditions. The distribution of final temperature is represented and an analysis is performed to compare the MF and HF cases. The effect of the machine parameters on the case depth is analyzed and the results are compared using experimental tests done on an induction machine.

## II. FINITE ELEMENT METHOD FORMULATION

Considering the electromagnetic material properties and neglecting hysteresis and magnetic saturation, the general equation governing the electromagnetic behavior is expressed in terms of magnetic potential vector ( $\mathbf{A}$ ) as [9].

$$\frac{1}{\mu(T)} \nabla^2 \mathbf{A} = -j\omega\sigma(T)\mathbf{A} + \mathbf{J}_0 \quad (1)$$

The resolution of the electromagnetic problem is used to calculate the thermal energy produced in the part during induction heating. The heat is generated in surface layer and transferred into the part core by conduction mode. The following equation shows that the heat can be expressed in function of the vector magnetic potential [9].

$$Q_{\text{Ind}} = \frac{\|(\nabla^2 \mathbf{A})^2\|}{\mu(T)\sigma(T)} \quad (2)$$

The thermal problem is coupled with the electromagnetic field analysis. The mode of heat transfer by conduction is most important during induction heating. The heat transfer action can be described by the following equation of Fourier-Kirchhoff. A neglected amount of created energy is lost at surface by convection and radiation modes. The convection is approximated to the conduction in the air surrounding the gear and the coil, and the radiation is neglected during heating due to the short heating time [9, 10].

$$k(T)\nabla^2 T = \gamma C_p(T) \frac{\partial T}{\partial z} + Q_{\text{Ind}} \quad (3)$$

## III. 3D FE SIMULATION

The gear heated by induction process was simulated with published physical data measured at thermodynamic equilibrium for the 4340 steel properties [11-13]. The simulation efforts were done using Comsol software (3D model). The simulation parameters considered for the modeling are the initial currents density in the coil ( $A.m^{-2}$ ), the heating time (s), and the frequency (kHz). The material properties behavior versus temperature had been evaluated in this study since finite element software can combine the required physical phenomena into a global model. The temperature dependence of the electrical conductivity, the relative magnetic permeability, the specific heat, the thermal conductivity and the coefficient of thermal expansion of the material have been considered in this study. These physical

properties are only known at thermodynamic equilibrium and transient phenomena taking place during fast heating or cooling are not taken into consideration. As the critical cooling rate to form martensite from austenite for the steel used in this research, work (4340 steel) was about 30 °C/s, the conciliator assumption was quite realistic in this case.

The developed model referred to a spur gear with an external diameter of  $\phi$  105 mm, a thickness of 6.6 mm and having 48 teeth, and a square section inductor made of copper. The material was regarded as homogeneous and isotropic. The FEM analysis took into account the following boundary limits: (1) the ambient temperature had been set to 293 °K, (2) the main components were surrounded by a local dielectric environment that is magnetically isolated along with vacuum permittivity and permeability. The convection was assumed equivalent to conduction in the air at interface due to the very short time of treatment. The loss of heat by radiation is neglected because of a very short time of heating.

An initial current density ( $J_0$ ) was applied at the inductor and it was varied to assess the temperature profile. The model represented a quarter of the tooth in order to reduce the computation time using symmetry conditions. The preliminary mesh used for the FE simulation is illustrated in Figure 1. The mesh was dense inside the part and the coil because of the high induced currents and temperature gradients between the surface and the core. A convergence study was conducted and it provides a model with optimized mesh.

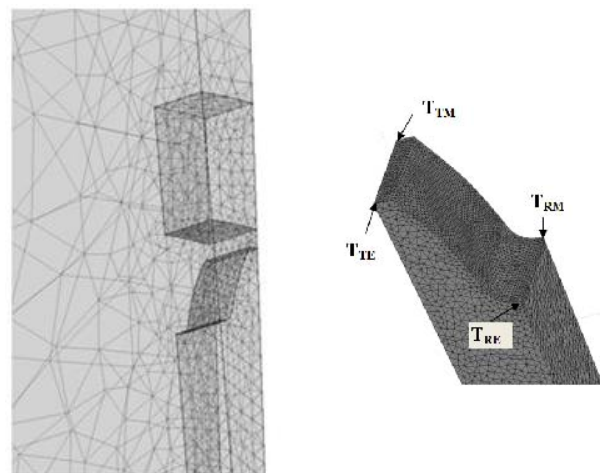


Fig. 1 Finite element method mesh

## IV. SIMULATION RESULTS

Two frequencies were considered in this study (10 kHz and 200 kHz). The heating time is fixed at 0.50 s. The heating time is very small since the heat treatment must be superficial and the part core should not be affected by the final temperature distribution. Moreover, the induction machine was very powerful and can transform part within 0.1 second. The initial current density ( $J_0$ ) in the coil was varied within a range permitting to have a desired surface temperature. Then, the simulated case depth is deducted at the edges and at the middle plan of the part for comparison and extract the effect of

$J_0$  and time. Figure 2 shows the distribution of the final temperature after a heating time of 0.5 s in the MF case (10 kHz). The  $J_0$  in the inductor is adjusted to  $2.15 \times 10^{10} \text{ A.m}^{-2}$  in order to have a maximal temperature of about 1000 °C at the end of heating. The heat generated in the part by the electromagnetic field had a direct relationship with the total current density.

The temperature distribution is a consequence of heat generated by the Joule effect. At first sight, root region was deeply heated in comparing to tooth tip. In this case, the temperature was more important at the edges and at the middle plan. Even if the temperature values at the surface are comparable, in depth they are slightly higher at the edges than in the middle plan. In this region, the case depth is very important and the edge effect is present. Then, these results conduct to non uniform final temperature across the root.

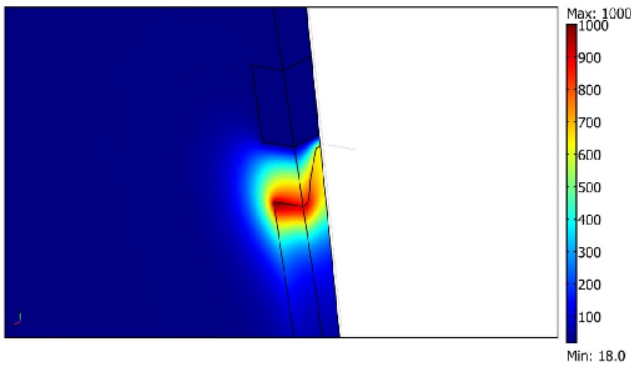


Fig. 2 Distribution of temperature (°C) (MF)

To better understand the evolution of heating, it is important to plot the temperature distribution at the edge and at the middle plan of the gear. At the edge, the root reaches high temperature after only 0.2 s and continues to increase to reach 1000 °C. At the middle plan, the temperature is less important than the edge plan and consequently, the case depth is less important at the edge upon cooling (Figure 3).

Figure 4 shows the distributions of the temperature after a heating time of 0.5 s in the HF case (200 kHz). The  $J_0$  is adjusted to  $3.8 \times 10^{10} \text{ A.m}^{-2}$  in the coil to also reach a maximal temperature of 1000 °C. The temperatures are higher at the part edges than at the middle plane and at the tooth tip. The transformation zones (the regions that have reached  $A_{c3}$ ) are deeper. These results show that the uniformity of the temperature distribution is a direct consequence of the induced currents and of the frequency. However, the root is transformed even if the high frequency since the magnetic field is strong at the edge. Consequently, the edge is transformed to martensite together with the tip.

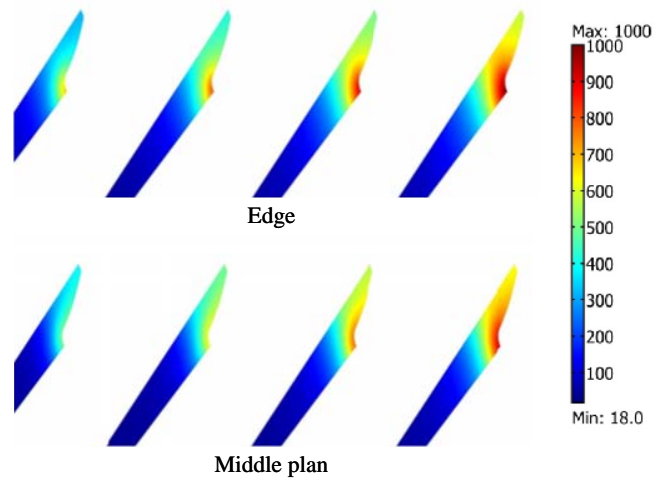


Fig. 3 Temperature distribution (°C) versus heating time (MF)

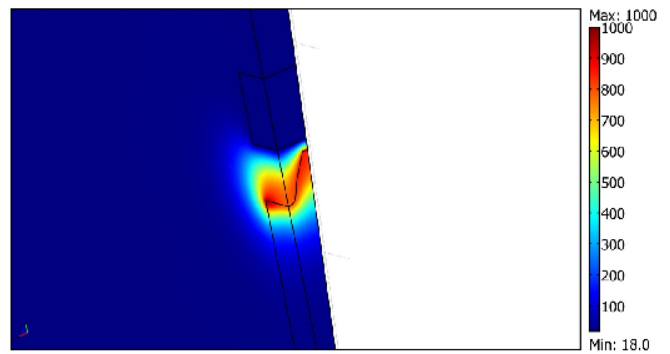


Fig. 4 Distribution of temperature (°C) (HF)

As illustrated in Figure 5, the temperature recorded at the part edges was higher than at the middle plane. After only 0.3 second, the temperature profile is uniform at the edge and only the tip is heated at the middle plan. With increasing time, the heat is more concentrated at the root than at the tip at the edge plan; while at the middle plan, the tip has the same temperature. The temperature difference between the both positions is more than 240 °C at the end of heating. This result is due to the electromagnetic edge effect that is caused by concentration of the magnetic field in region with drastic geometrical change. Thus, one can early conclude that the case depth is larger at edges than middle plan at the root and it is slightly the same at the tip.

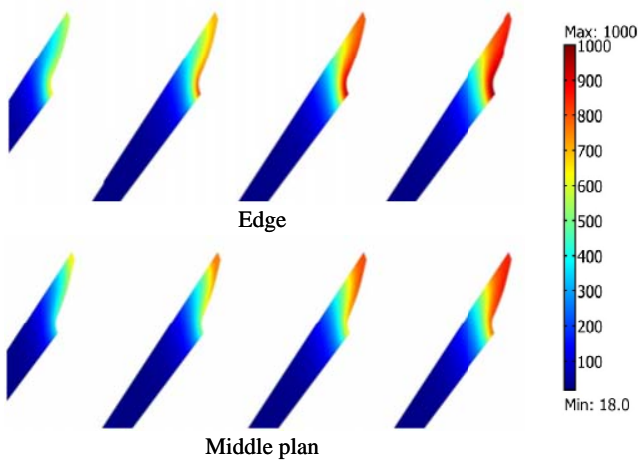


Fig. 5 Temperature distribution (°C) versus heating time (HF)

The obtained results demonstrate the presence of edge effect in both MF and HF gear heating unlike the simple geometries previously treated in the literature [14]. The figure 6 presents the evolution of temperatures curves with time in the MF case. The four temperatures considered were:  $T_{RE}$  and  $T_{RM}$  at the root at the edge and the middle plan, and  $T_{TE}$  and  $T_{TM}$  at the tip at the edge and the middle plan. The obtained results demonstrate that the temperature  $T_{RE}$  is more important and has an offset compared to the temperature at the root at the middle plan. However, the temperature at the tip is less important and the results confirm that this region doesn't transform to martensite. While the assumption stipulating that all regions heated above the  $A_{c3}$  temperature become hard martensite after cooling is met and that this temperature is estimated at 800 °C for heating rates involved, it is possible to determine easily the case depth. The figure 7 presents the same temperatures at the tooth tip. The temperatures exceed the austenitizing temperature at the tip and at the root (edge). Consequently, these regions will be transformed to hard martensite.

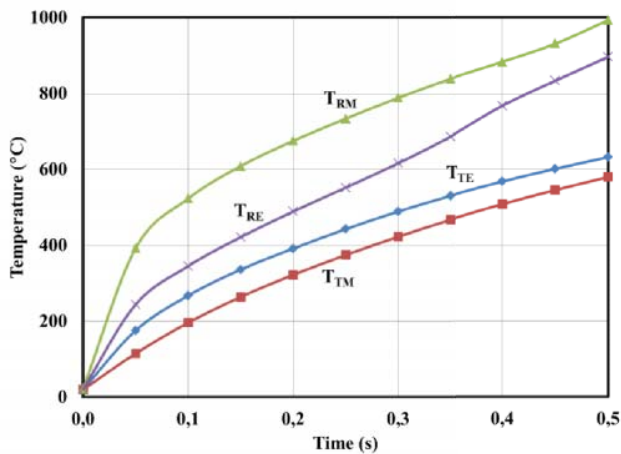


Fig. 6 Temperature versus time (MF)

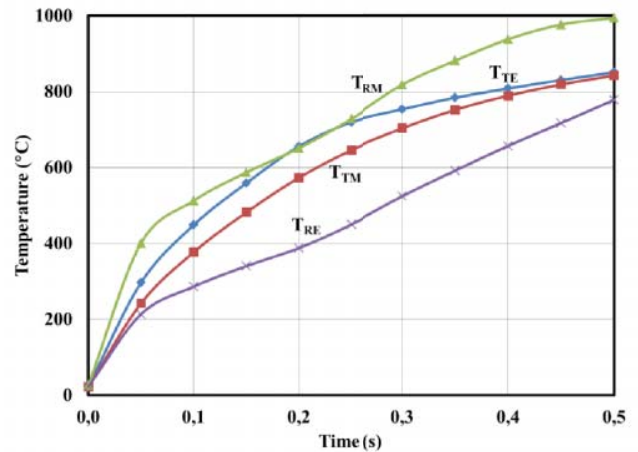


Fig. 7 Temperature versus time (HF)

### V. EXPERIMENTAL VALIDATION

Since, it is not easy to measure the practical quantities such as current in the coil or surface temperature and to give the gap between simulation results and experimental tests, it is necessary to develop a new method able to reduce or eliminate these differences by adjusting the simulation parameters. First, it is important to adjust  $J_0$  in order to obtain the reasonable temperatures and obviously the proper hardness profiles.

To validate the developed numerical models, it is then necessary to perform experimental tests at MF and HF by modulating the time and the machine power ( $P_M$ ) and using the simulation to reach the same hardness profile. The validation tests results show a clear concordance between the simulation and the experiments (Figures 8, 9, 10 and 11).

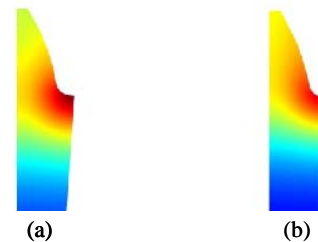


Fig. 8 Temperature profile obtained by simulation at (a) Edge and (b) Middle plan (MF) [14]

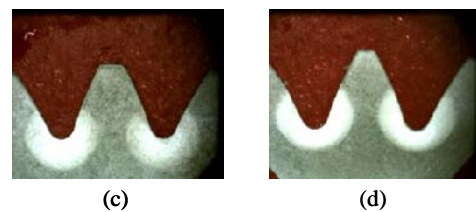


Fig. 9 Hardness profile obtained by experimental tests at (a) Edge and (b) Middle plan (MF) [14]



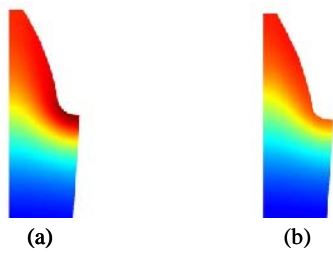


Fig. 10 Temperature profile obtained by simulation at (a) Edge and (b) Middle plan (HF)

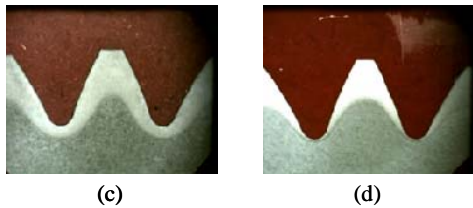


Fig. 11 Hardness profile obtained by experimental tests at (a) Edge and (b) Middle plan (HF) [14]

It is pertinent to note that the mean power provided to the part during heating process can be evaluated by simulation. This power ( $P_S$ ) can be represented as a linear function of the initial currents density ( $J_0$ ). During simulation development, it is practical to choose the power  $P_S$  for heating and then retrieve the initial current density to use as a parameter before to conduct the computation. Consequently, ratios between  $P_M$  and  $P_S$  can be determined for each test. These ratios represent a practical and useful correlation between the both preceding powers to help the induction heating engineers to choose recipes for parts under various conditions. For each done test, the initial currents density is adjusted using simulation to approach the real case depth.  $P_S$  is then calculated and the power ratio is established. The obtained results show that there is less power loss in the HF case compared to MF case and the ratio increases lightly with heating time. It is important to note that the power in the MF case is twice higher than in HF as shown in table 1.

TABLE I  
 POWER RATIOS

Test	$P_M$ (kW)	$P_S$ (kW)	Ratio
1 (MF)	220	153	65 %
2 (HF)	83	65	80 %

## VI. CONCLUSION

The paper has presented an analysis of edge effect using 3D finite element simulation and validation applied to spur gear

heated by induction. First, a 3D model has been developed by simulation. Second, a complete comparison between the MF and HF heating cases was conducted to emphasize the electromagnetic and thermal effects. The obtained results show that the simulation represents a powerful tool to better understand how the currents and the temperatures are distributed in the part and how they are affected by the material properties behavior. They are useful also to understand where the heat energy is generated during the heating process. The experimental validation and calibration gave the power ratios between the input machine power and simulation power required to achieve quantitative prediction. The MH and HF have been studied separately and without preheating flash. An analysis of the effect of preheating and dwell are under investigation and will be the topic of future publications.

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