Photon Localization inside a Waveguide Modeled by Uncertainty Principle

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***Abstract*—**In the present work, an attempt is made to understand electromagnetic field confinement in a subwavelength waveguide structure using concepts of quantum mechanics. Evanescent field in the waveguide is looked as inability of the photon to get confined in the waveguide core and uncertainty of position is assigned to it. The momentum uncertainty is calculated from position uncertainty. Schrödinger wave equation for the photon is written by incorporating position-momentum uncertainty. The equation is solved and field distribution in the waveguide is obtained. The field distribution and power confinement is compared with conventional waveguide theory. They were found in good agreement with each other.

***Keywords*—**photon localization in waveguide, photon tunneling, quantum confinement of light, Schrödinger wave equation, uncertainty principle.

I. Introduction

RECENTLY there has been a growing interest in reconstructing the quantization description of photon. Some recent studies have shown that photons can be localized in space [1-3]. It is argued that the photon tunneling process originates in an inability to localize photons completely in space.

According to Ole Keller [4], when light propagates through subwavelength structures, it is far beyond the classical diffraction limits. Hence, field confinement in this case can be explained by concepts of quantum mechanics. The first-quantized theory of spatial localization of photons is proposed by Ole Keller, in which the spatial field localization is smoothly linked to classical electromagnetic theory. Photon wave function is also formulated and used to explain photon propagation in the media. Photon localization is explained by constructing photon wave functions [5-10]. It is shown that a single photon cannot be localized in the same sense that a massive particle in nonrelativistic quantum mechanics can be localized [11]. However, Ole Keller has made following arguments regarding photon localization as given in [12]. According to Keller, not only the massless particle such as the photon, but also the massive particle itself, cannot be localized since, when a particle is confined to a small region, nonrelativistic quantum mechanics may become no longer valid. Hence, concept of localizability in nonrelativistic quantum mechanics is just an ideal.

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On the other hand, in relativistic quantum mechanical picture, there exists a common property for the localizability of all massive particles. It is further argued that it is impossible to localize a particle with a greater precision than its Compton wavelength, which is because of many-particle phenomena. Thus, even though there has been no nonrelativistic quantum mechanics of the photon, localizability of the photon can be still studied in the sense that a massive particle in relativistic quantum mechanics can be localized.

In practice, an interaction is necessary to locate a particle. The photon propagating in a waveguide provides the required interaction. Thus, photons inside a waveguide can be localized in the same sense that a massive particle can be localized in free space. It is argued that tunneling photons can be regarded as evanescent modes and can propagate over a spacelike interval [13]. Thus evanescent modes are a quantum mechanical rather than a classical phenomenon.

This has been the motivation for our work. In the present work, attempts are made to understand light confinement in a subwavelength planar waveguide structure in terms of probability of photon localization in the waveguide structure.

II. Our idea of understanding photon localization

Light confinement in a waveguide structure is well explained with Maxwell's equations. Existence of evanescent field originates from condition of total internal reflection required for light wave propagation inside the waveguide. However, when the light wave with wavelength greater than waveguide dimension propagates through the waveguiding structure, the light propagation and total field distribution can be better understood in terms a photon localized in the waveguide with a finite tunneling probability through the waveguide core. So it becomes a quantum mechanical phenomenon. It is known that electron tunneling is well described by concepts of quantum mechanics. In quantum mechanical picture, a photon and an electron can be treated analogously [14]. They exhibit many similar characteristics. Electron tunneling has exponentially decaying amplitude in classically forbidden region. Thus, evanescent field in the cladding of a waveguide which decays exponentially can be looked as a barrier tunneling phenomenon, in quantum mechanical regime. Barrier tunneling of electrons is explained by Schrödinger wave equation.

In present work, attempts are made to obtain Schrödinger wave equation (SWE) for the waveguide photon. Waveguide modes are obtained by solving the Schrödinger wave equation with appropriate boundary conditions.

III. Schrödinger Wave Equation

Solution of Schrödinger wave equation for electron tunneling shows a finite probability of transmission across the barrier. Since it is probability phenomenon, it can be understood in terms of Heisenberg's uncertainty principle. Angik Sarkar and T. K. Bhattacharyya [15] have well explained barrier tunneling of electron in terms of Heisenberg's uncertainty principle as follows. When a packet of electrons is incident at a potential barrier of some finite height and if their position uncertainty Δx is greater than width of the barrier, then there is a definite probability of some electrons being on the other side of the barrier. Momentum uncertainty is calculated for a given value of Δx, from which energy values are obtained. This analysis gives a good justification to the experimental data.

For understanding photon localization in a subwavelength waveguide and existence of evanescent field, we extend the above arguments made for an electron to the photon problem. We look at the field distribution inside waveguide in the form of propagating field and outside the waveguide in the form of evanescent field as probability phenomenon and use uncertainty principle to describe the field distribution. The field distribution is a result of momentum conservation in the three regions of the waveguide. Hence the energy distribution in the three regions is obtained in terms of momentum uncertainties in the three regions.

The Schrödinger wave equation for an electron as given in (1) is modified as that given in (2) & (3), as follows:

$$\frac{\partial^2\Psi}{\partial x^2}+\frac{2m}{\hbar^2}(E-V)\Psi=0 \tag{1}$$

where symbols have their usual meaning.

We write the Schrödinger wave equation for the photon as:

$$\frac{\partial^2\Psi}{\partial x^2}+\frac{4\pi\mu_{core}{}^2}{\lambda_o\Delta x}\Psi=0\text{ ; for } 0\le x\le a \tag{2}$$

$$\frac{\partial^2\Psi}{\partial x^2}-\frac{4\pi\mu_{core}{}^2}{\lambda_o\Delta x}\Psi=0\text{ ; for } -b\le x\le 0\text{; and } a\le x\le a+b \tag{3}$$

where 'a' is the waveguide height and 'b' is the upper and lower cladding height. The term $\Delta\mu$ represents difference in refractive indices of core and cladding.

Equation (2) is derived by using electron-photon analogy in Schrödinger wave equation as given in (1). Equation (2) is obtained by using position momentum uncertainty principle for the photon. The change in momentum is accounted for by change in velocity or ultimately by change in refractive index in the cladding regions.

IV. Comparison of Results Obtained by Conventional Theory and the Present Work

Three layer planar waveguide structure is analyzed to validate our proposed Schrödinger wave equation for a waveguide photon by calculating power in core and obtaining its dependence on propagating wavelength and waveguide height. The waveguide has silicon dioxide (SiO_2) as the lower cladding or substrate, Silicon (Si) core and air as (upper) cladding.

A. Conventional theory

For waveguide analysis by conventional theory, Maxwell's equations are written as given in [16] for a TE mode condition. Single mode waveguide is used for analysis. The eigen value equation as given in (4) is solved numerically for obtaining axial propagation constant (β) and fraction of power in core (η).

$$\tan(\kappa h)=(\gamma_{air}+\gamma_{SiO_2})/(\kappa(1-\gamma_{air}\gamma_{SiO_2}/\kappa^2)) \tag{4}$$

Here, $\gamma_{air}=\sqrt{\beta^2-k_o{}^2\mu_{air}{}^2}$, $\gamma_{air}=\sqrt{\beta^2-k_o{}^2\mu_{SiO_2}{}^2}$,

k_o is the wave vector in air and κ is transverse wave vector in the waveguide.

Power in the waveguide is calculated by using (5) as follows.

$$P=\frac{\beta}{2\omega\mu_0}\int_{-\infty}^{\infty}\left|E_y\right|^2 dx \tag{5}$$

Fraction of power in core or confinement factor is calculated as $\eta = P_{core}/P$.

B. Proposed theory

Present work is focused on understanding photon localizability in a waveguide. Hence, field confinement in the waveguide core is studied. For that, Schrödinger wave equations as given by (2) and (3) are solved numerically to find field confinement factor. Appropriate boundary conditions for continuity of field vector across the core-cladding boundaries are used. Equation (2) is used for core region and equation (3) is used for the two claddings. The power in core in this case is calculated by using the expression $\eta = P_{core}/P$., where P and P_{core} are given by (6) & (7)

$$P=\int_{-b}^{a+b}\left|\psi\right|^2 dx \tag{6}$$

$$P_{core}=\int_{0}^{a}\left|\psi\right|^2 dx \tag{7}$$

Where wave vector ψ is obtained by numerically solving (2) and (3)

V. Results

Fraction of power in core is calculated by using both the theories. Dependence of power in core on propagating wavelength and waveguide height is studied. The analysis of planar Air-Si-SiO_2 waveguide structure yields following results as depicted in Fig. 1 and Fig. 2.

A. Dependence on wavelength

Fractional power in core for various wavelengths is calculated for a waveguide of height of 220nm.

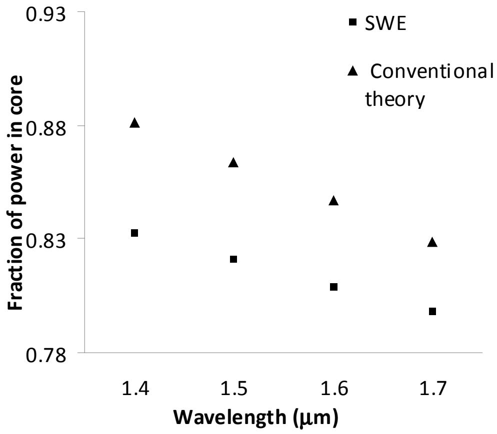

Fig. 1 Comparison of field confinement for various propagating wavelengths obtained by conventional theory and Schrödinger wave equation for planar waveguide of height 220nm

B. Dependence of fractional power in core on waveguide height

Fractional power in core is calculated for various waveguide heights for 1.55µm wavelength.

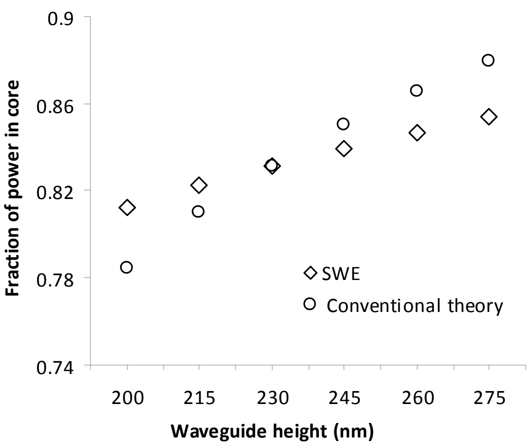

Fig. 2 Comparison of field confinement for various waveguide heights obtained by conventional theory and Schrödinger wave equation

From Fig. 1, it is observed that the trends and values of fractional power in core with wavelength obtained from Schrödinger wave equation almost matches with conventional theory.

From Fig. 2 also, trends and values of fractional power in core with waveguide height obtained from Schrödinger wave equation and conventional theory are found in good agreement with each other.

VI. Conclusion

Schrödinger wave equation is obtained for a photon in waveguide using by quantum mechanical concepts. It is found helpful in obtaining field confinement in a subwavelength waveguide structure. In this work, we have made an attempt to understand photon localization in the waveguide. This work gives a different approach to look at the field confinement problem, which can be explored further.

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