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Slip Suppression of Electric Vehicles using Model Predictive PID Controller

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Abstract—In this paper, a new model predictive PID controller design method for the slip suppression control of EVs (electric vehicles) is proposed. The proposed method aims to improve the maneuverability and the stability of EVs by controlling the wheel slip ratio. The optimal control gains of PID framework are derived by the model predictive control (MPC) algorithm. There also include numerical simulation results to demonstrate the effectiveness of the method.

Keywords—Model Predictive Control, PID controller, Electric Vehicle, Slip suppression

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I. Introduction

ELECTRIC vehicles (EVs) have received much attention in recent years as a countermeasure to global warming and for being Eco friendly [1], [2], [3], [4].

EVs are automobiles which are propelled by electric motors, using electrical energy stored in batteries or another energy storage devices. Electric motors have several advantages over (internal-combustion engines) ICEs:

- Energy efficient. Electric motors convert 75% of the chemical energy from the batteries to power the wheels - ICEs only convert 20% of the energy stored in gasoline.
- Environmentally friendly. EVs emit no tailpipe pollutants, although the power plant producing the electricity may emit them. Electricity from nuclear-, hydro-, solar-, or wind-powered plants causes no air pollutants.
- Performance benefits. Electric motors provide quiet, smooth operation and stronger acceleration and require less maintenance than ICEs.
- Reduce energy dependence. Electricity is a domestic energy source.

The travel distance per charge for EV has been increased through battery improvements and using regeneration brakes, and attention has been focused on improving motor performance. The following facts are viewed as relatively easy ways to improve maneuverability and stability of EVs.

1) The input/output response is faster than for gasoline/diesel engines.
2) The torque generated in the wheels can be detected relatively accurately
3) Vehicles can be made smaller by using multiple motors placed closer to the wheels.

Much research has been done on the stability of general automobiles, for example, ABS (Anti-lock-Braking Systems), TCS (Traction-Control-Systems), and ESC (Electric-Stability-Control)[5] as well as VSA (Vehicle-Stability-Assist)[6] and AWC (All-Wheel-Control) [7]. What all of these have in common is that they maintain a suitable tire grip margin and reduce drive force loss to stabilize the vehicle behavior and improve drive performance. With gasoline/diesel engines, however, the response time from accelerator input until the drive force is transmitted to the wheels is slow and it is difficult to accurately determine the drive torque, which limits the vehicle's control performance.

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This paper deals with the slip suppression of EV for controlling the wheel slip ratio. Conventional gasoline/diesel vehicles are equipped with a TCS, which requires expensive sensors and additional equipment, but, as mentioned above, EV have a fast torque response and the motor characteristics can be used to accurately determine the torque, which makes it relatively easy and inexpensive to realize high-performance traction control. This is expected to improve the maneuverability and stability of EV. It's, therefore, important to research and development to achieve high-performance EV traction control with slip suppression. Various proposals have been made for EV traction control, such as a system based on motor torque current dropping characteristics[8], a system that utilizes a nonlinear controller[9], and a system that controls the slip ratio with wheel control [10], but this paper proposes a PID control method based on model predicative control (MPC).

PID controllers have a simple construction and have been proved to be practical and highly reliable in many industrial fields[11], [12]. The proposed method determines the PID controller gain using an MPC algorithm to utilize the capability of explicitly considering the constraints, which is one of the advantages of MPC, to achieve a more advanced and flexible control method[13], [14], [15]. Specifically, the optimum control input is calculated by the MPC explicitly considering the constraints and the PID gain for realizing this is derived in advance newline and used. For this reason, various road surfaces the vehicle will travel on are presumed, the optimum PID for each of them derived, and the creation for a look up table for these is envisioned. During actual driving, the values of PID gains in the look up table are referenced according to the conditions to be able to support a vehicle with fast-moving dynamic characteristics. This is based on the explicit-MPC concept[16]. The advantages of this method are that the structure is simple, that is uses a proved and reliable PID controller, and that it achieves flexible, high-performance control by explicitly considering the constructions the same as for MPC. Numerical examples are used to compare the proposed method to conventional methods and to verify the effectiveness of the proposed method.

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II. Electric Vehicle Dynamics

As a first step toward practical application, this paper restricts the vehicle motion to the longitudinal direction and uses direct motors for each wheel to simplify the one-wheel model to which the drive force is applied. In addition, braking was not considered this time with the subject of the study being limited to only when driving.

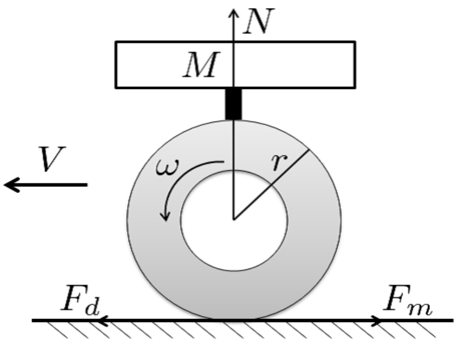

Fig. 1. One-wheel car model

From fig. 1, the vehicle dynamical equations are expressed as eqs. (1) to (3).

$$M\frac{dV}{dt} = F_d(\lambda) - F_a - \frac{T_r}{r} \quad (1)$$

$$J\frac{d\omega}{dt} = T_m - rF_d(\lambda) - T_r \quad (2)$$

$$F_m = \frac{T_m}{r} \quad (3)$$

$$F_d = \mu(c,\lambda)N \quad (4)$$

Where M is the vehicle weight, V is the vehicle body velocity, F_d is the driving force, J is the wheel inertial moment, F_a is the resisting force from air resistance and other factors on the vehicle body, T_r is the frictional force against the tire rotation, ω is the wheel angular velocity, T_m is the motor torque, F_m is the motor torque force conversion value, r is the wheel radius, and λ is the slip ratio. The slip ratio is defined by (5) from the wheel velocity (V_ω) and vehicle body velocity (V).

$$\lambda = \begin{cases} \dfrac{V_\omega - V}{V_\omega} & \text{(accelerating)} \\ \dfrac{V - V_\omega}{V} & \text{(braking)} \end{cases} \quad (5)$$

λ during accelerating can be shown by (6) from fig. 1.

$$\lambda = \frac{r\omega - V}{r\omega} \quad (6)$$

The frictional forces that are generated between the road surface and the tires are the force generated in the longitudinal direction of the tires and the lateral force acting perpendicularly to the vehicle direction of travel, and both of these are expressed as a function of λ. The frictional force generated in the tire longitudinal direction is expressed as μ, and the relationship between μ and λ is shown by (7) below, which is a formula called the Magic-Formula[17] and which was approximated from the data obtained from testing.

$$\mu(\lambda) = -c_{road} \times 1.1 \times (e^{-35\lambda} - e^{-0.35\lambda}) \quad (7)$$

Where c_{road} is the coefficient used to determine the road condition and was found from testing to be approximately $c_{road} = 0.8$ for general asphalt roads, approximately $c_{road} = 0.5$ for general wet asphalt, and approximately $c_{road} = 0.12$ for icy roads. For the various road conditions ($0 < c < 1$), the $\mu - \lambda$ surface is shown in fig. 2. It shows how the friction

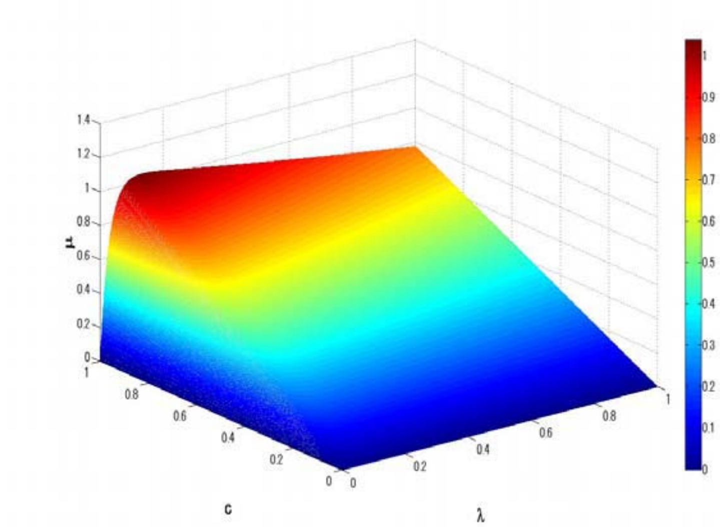

Fig. 2. λ-μ surface for road conditions

coefficient μ increases with slip ratio λ ($0.1 < \lambda < 0.2$) where it attains the maximum value of the friction coefficient. As defined in (**??**), the driving force also reaches the maximum value corresponding to the friction coefficient. However, the friction coefficient decreases to the minimum value where the wheel is completely skidding. Therefore, to attain the maximum value of driving force for slip suppression, it should be controlled the optimal value of slip ratio. the optimal value of λ is derived as follows.

Choose the function $\mu_c(\lambda)$ defined as

$$\mu_c(\lambda) = -1.1 \times (e^{-35\lambda} - e^{-0.35\lambda}). \quad (8)$$

By using (8), (7) can be rewritten as

$$\mu(c,\lambda) = c_{road} \cdot \mu_c(\lambda). \quad (9)$$

Evaluating the values of λ which maximize $\mu(c,\lambda)$ for different $c(c > 0)$, means to seek the value of λ where the maximum value of the function $\mu_c(\lambda)$ can be obtained. Then let

$$\frac{d}{d\lambda}\mu_c(\lambda) = 0 \quad (10)$$

and solving equation (10) gives

$$\lambda = \frac{log100}{35 - 0.35} \approx 0.13. \quad (11)$$

Thus, for the different road conditions, when $\lambda \approx 0.13$ is satisfied, the maximum driving force can be gained. Namely, from (7) and fig. 2, we find that regardless of the road condition (value of c), the $\lambda - \mu$ surface attains the largest value of μ when λ is the optimal value 0.13.

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III. Model Predictive PID Control for Slip Suppression

A. Problem Formualation

From (11) and discussion above, to transmit the wheel drive force to the road surface with as little loss as possible it is best to maintain the value of λ at around 0.13.

However, to maintain the value of λ at around 0.13 under various changing conditions is practically impossible.

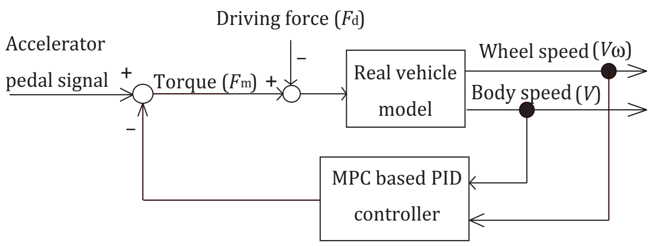

Fig. 3. Block diagram of model predictive PID control system

In this paper, therefore, as is shown in fig. 2, the value of λ is kept within the range of $0.05 \leq \lambda \leq 0.20$, in which the drive force loss is not expected to so large, with the practical goal of maintaining the value as close to 0.13 as possible. The model predictive PID control proposed in the next section is used to configure the control system shown in Fig. 3 to perform traction control by controlling the slip ratio.

In the MPC based PID control, the control input which is nearly equivalent to the one nbased on the MPC is offline-calculated in advance. In other words, it's the explicitly determining method of the value of PID controller gains. It can also be said to be an explicit-MPC method under the restriction to the PID controller structure. The computation time that is a disadvantage of standard MPC can be shortened by this. Furthermore, by using the PID controller structure, that is currently being used in many various control systems, it can be promised for the configuration of highly reliable and practical control systems.

B. MPC

MPC had a major impact during the late 1970s mainly as the control method for industrial processes, and one of its characteristics is the ability to implicitly consider the constraints relating to devices, etc. A conventional MPC, however, must be able to repeatedly make the optimum calculation online, so it was only applied when the control subject was a chemical or other system with slow operating characteristics, but recent improvements in computer performance is making possible increasing application in mechatronics and other areas. MPC predicts the future quantity of state behavior from the current quantity of state based on a control subject model, and of the control inputs obtained by solving the optimization problem in the finite interval based on the set constraints, only the first is considered the control subject and repeated. Specifically for instance, we can consider the discrete-time state space model below.

$$x(k+1) = Ax(k) + Bu(k) \tag{12}$$

$$y(k) = Cx(k) \tag{13}$$

There are several objective functions for MPC, but here I use the most general LQ (Linear Quadratic) criteria, and specifically (14), which uses weighted matrices Q and R, as the objective function.

$$J = \sum_{i=0}^{H_p} \| x(k+i) - r(k+i) \|_Q^2 + \sum_{i=0}^{H_u-1} \| \Delta u(k+i) \|_R^2 \tag{14}$$

In this case, MPC finds

$$\Delta U_{opt}(k) := \left(\Delta u_{opt}(k), \quad \cdots, \quad \Delta u_{opt}(k+H_u-1) \right)^T$$

applied to the minimum value of J at the current time k such that $u(k)_{opt} := \Delta u(k)_{opt} + u(k-1)$, and this is input into the control object before moving to the next step and repeating the process. H_p and H_u are the prediction interval and control interval, respectively, and these predict the model behavior at a certain number of steps ahead of the current time, or calculates the control input a certain number of steps ahead. We know that $\Delta U(k)_{opt}$ is obtained from $\nabla J = 0$, so calculating this to find $u(k)_{opt}$ as follows[13].

$$\Delta u(k)_{opt} = [BQB+R]^{-1} BQ(r - Ax(k) - Bu(k-1)) \tag{15}$$

C. Model predictive PID controller

PID is an acronym created from Proportional (proportional action), Integral (integral action), and Derivative (derivative action), and it has a simple structure that makes it easy to intuitively understand the role of each action and thus has been used for many years in a variety of fields and today remains a proved, highly reliable control device used for a variety of subjects.

The continuous-time system PID controller control input is generally expressed by (16).

$$u(t) = K_P e(t) + K_I \frac{1}{T_I} \int_0^t e(\tau) d\tau + K_D \frac{de(t)}{dt} \tag{16}$$

Where $e(t) := r(t) - y(t)$ (deviation), and where K_P, K_I, and K_D are called the proportional gain, integral gain, and differential gain, respectively. The MPC control input mentioned in the previous section expressed using the Δu of the discrete-time system, so an effort was made to match this. First, for time $t = k$, it's expressed by

$$u(k) = K_P e(k) + K_I \sum_{i=0}^{k} e(i) + K_D(e(k) - e(k-1)) \tag{17}$$

Then, we can calculate the Δu used by (17) as follows.

$$\begin{aligned} \Delta u &= u(k) - u(k-1) \\ &= K_P \Big(y(k) - y(k-1) \Big) \\ &\quad + K_I \Big(r(k) - y(k) \Big) \\ &\quad + K_D \Big(-y(k) + 2y(k-1) - y(k-2) \Big) \end{aligned} \tag{18}$$

Therefore, the design problem of model predictive PID controller is as follows.

$$\min_{K_P,K_I,K_D} J_{pid} \tag{19}$$

$$subject\ to \quad u_{min} \leq u(k+i) \leq u_{max} \tag{20}$$

where

$$J_{pid} := \sum_{i=0}^{H_p-1} \left(\| e(k+i+1) \|_Q^2 + \| \Delta u(k+i) \|_R^2 \right)$$

and where u_{min} and u_max are the minumum and maximum input respectively.

IV. Numerical Experiments

A. Experimental setup

As shown by eqs. (1) $\sim$ (4), the vehicle model has nonlinear characteristics and has been difficult to control design as it is. up to In this paper, therefore, I consider a linear approximated model as the perturbed system in the time ($t = k$) used. If we use the slip ratio in the time $t = k$ as λ_k, and the $\lambda - \mu$ curve inclination in λ_k as

$$a = \frac{d\mu}{d\lambda}\Big|_{\lambda_k}, \tag{21}$$

and using eqs. (1) $\sim$ (4), the relation of variation of the slip ratio $\Delta\lambda$ and variation of the motor torque ΔT_m is expressed as follows.

$$\frac{\Delta\lambda}{\Delta T_m} = \frac{M(1-\lambda_k)}{aN\left[M(1-\lambda_k) + \frac{J}{r^2}\right]} \times \frac{1}{\tau_a s + 1} \tag{22}$$

where

$$\tau_a := \frac{J\omega M(1-\lambda_k)}{arN\left[M(1-\lambda_k) + \frac{J}{r^2}\right]} \tag{23}$$

The transfer function is numerically realized using the application software, $Matlab(Ver.7.5.0.342)$ and $Simulink(Ver.6.3)$ as the state space of the SISO (Single Input Single Output) system. Note that for a one wheel model, K_D is normally 0, and thus is the PI control system in this case.

B. Simulation results

As an example, the effectiveness of the proposed method is verified by numerical simulations of start off performance on icy roads. First, the vehicle parameter values are shown in table I. By using the values in Table I, the derived values of the model predictive PID controller gains when torque is applied for 60 sec. at approximately 55 [km/h] on a icy road are shown in figs. 4 and 5.

Where the constraints for the variation of torque upper and lower limits are set to $\Delta u_{max} = 100[Nm]$ and $\Delta u_{min} = -200[Nm]$. This is done because for an actual vehicle to avoid an unintentional risky situation where the driver cannot control the vehicle because excessive torques change is possible. Normally, it is desirable for the variation of torque upper and lower limits to be experimentally set from the driver accelerator depression amount practical data, etc., but here for convenience this has been set using these values. Note that the objective function weighted Q and R in the proposed method were set using trial and error.

Next, the obtained value was used to configure the model predictive PID control system and the results from the conducted simulation are shown below. fig. 7 shows the change in slip ratio during 0 to 10 sec., figs. 8 and 9 show the change in vehicle body speed and wheel speed during 0 to 10 sec., and fig. 10 shows the change in motor torque during 0 to 10 sec.

For comparison, the results with no control and with

TABLE I
VALUES OF CAR PARAMETERS

M:Mass of vehicle	$1100[kg]$
J :Inertia of wheel	$21.1[kg/m^2]$
r: Radius of wheel	$0.26[m]$
g: Gravitational acceleration	$9.81[m/s^2]$

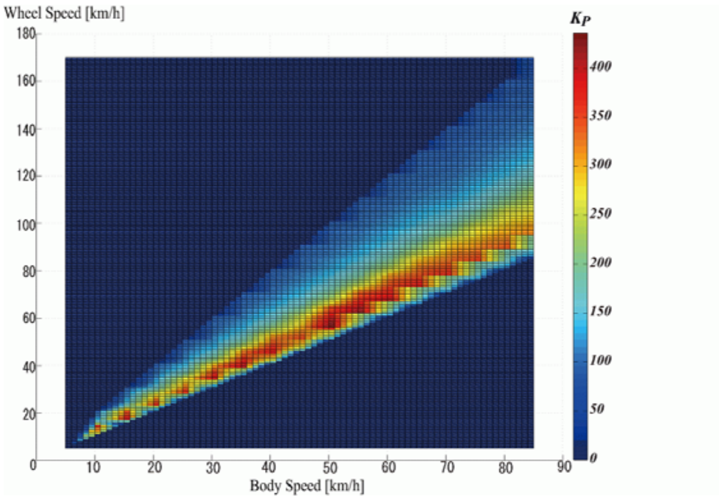

Fig. 4. Proportional gain (K_P)

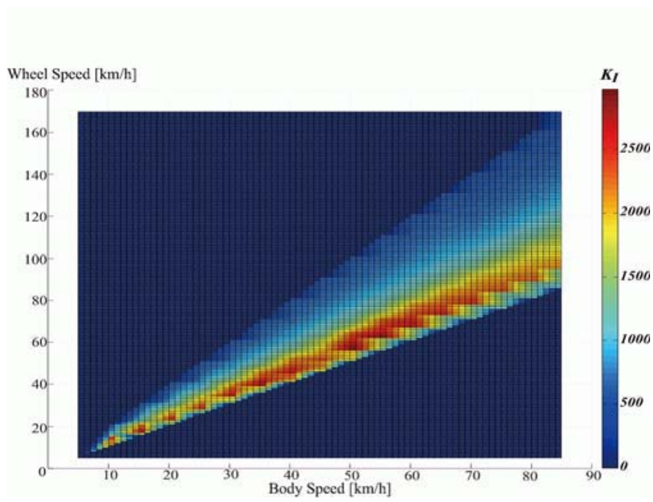

Fig. 5. Integral gain (K_I)

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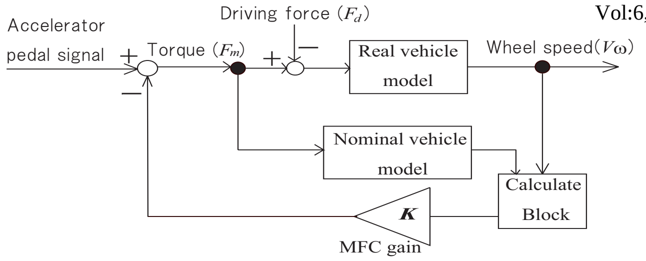

Fig. 6. Block diagram of model following control system

the conventionally MFC [18] are also shown. The MFC is a control method that uses the ideal state model with no wheel slip (the wheel and road surface are adhered together) as the nominal model and uses the proportional feedback based on the difference between the actual wheel speed and the nominal wheel speed. As shown in fig. 6, the quantity of state used is only in the wheel speed, which has the advantage of making possible a simple configuration and control with good response and not requiring the vehicle body speed to be observed.

First, when not conducting control, we can see from fig. 7 that at the initial stage the slip ratio increases rapidly and tire spinning occurs. The tire spinning continues thereafter as well since a system for reducing the drive force to suppress the tire spin is not employed, and so driving continues without reducing the slip ratio λ. This results in much loss of the drive force that is transmitted to the road surface and show that tire spin is always occurring, which results in little increase in vehicle body speed compared to the rapid increase in wheel speed shown in figs. 8 and 9 and thus prevents the vehicle from accelerating.

Next, in a vehicle employing MFC, there is a temporary large increase during the initial stage (low-speed range) as shown in fig. 7, but the slip ratio is being gradually suppressed. This is due to the torque reduction effect from the proportional feedback control from the comparison with the nominal model. Since MFC uses a vehicle in an ideal state where there is no slip as the nominal model, following

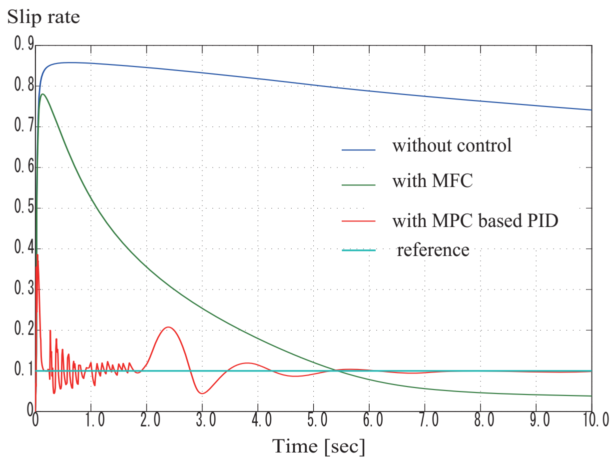

Fig. 7. Slip ratio

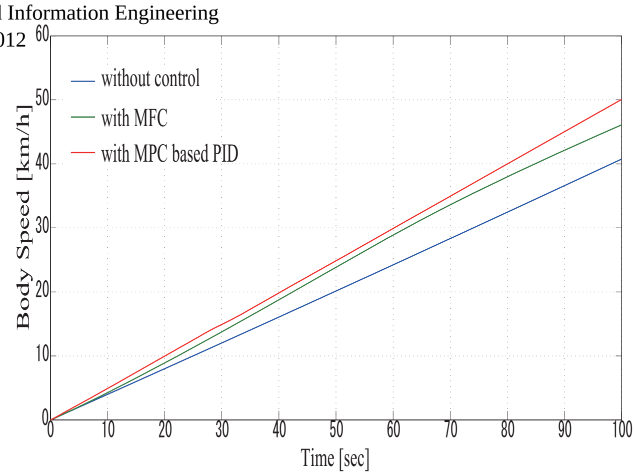

Fig. 8. Vehicl velocity

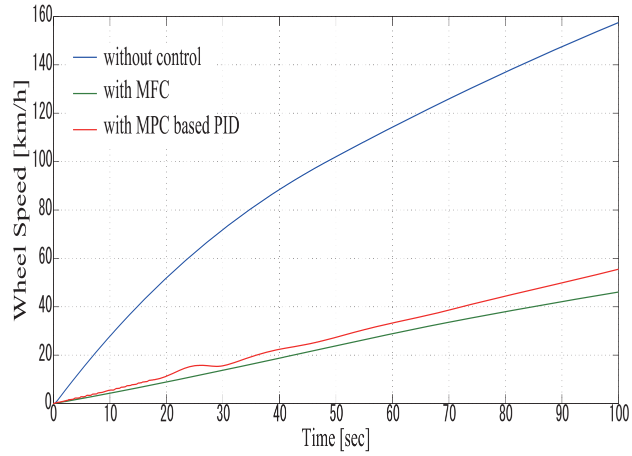

Fig. 9. Body and wheel velocities

a characteristic slip ratio of $\lambda = 0$ is used as the target. Therefore, after about 50 sec. λ is 0.1 or less. For this reason this results in the loss of drive force, and sufficient acceleration performance compared with the proposed method cannot be obtained as shown in figs. 8 and 9.

In a vehicle employing the proposed method, the slip ratio increase is sufficiently suppressed in the initial stage as well as is shown in fig. 7. In addition, the overall target of $0.08 \leq \lambda \leq 2.00$ is almost attained, and the highest torque efficiency of 0.1 is followed with good speed response. In fig. 7, the waveform differs between 0 to 2.0 sec. and 2.0 to 5.0 sec., this is due to the PID gain setting based on the prediction for the set prediction interval by the MPC and is caused by the changing timing as the gain value shifts as the optimum value for the prediction interval width and evaluation function is sought during the transition from the rapid change during start off in particular to the stabilized situation during steady state.

As shown in fig. 10, the proposed method sufficiently utilizes the change in input torque while performing good control to suppress tire spin and achieves efficient wheel and vehicle body acceleration as shown in figs. 8 and 9. Note that the rapid fluctuation in the slip ratio and torque in the

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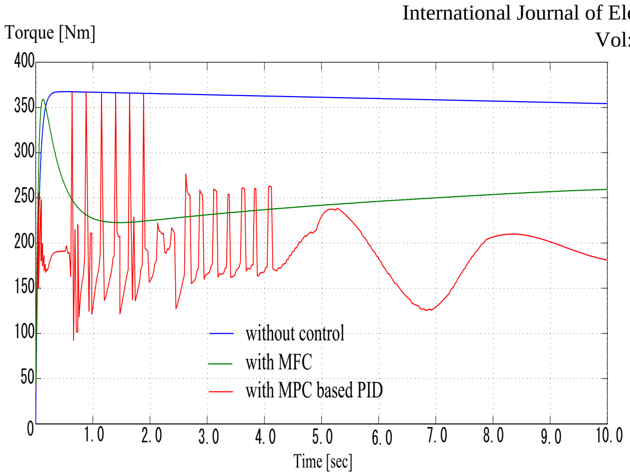

Fig. 10. Motor torque

low-speed range is caused by referring to the look up table at a sampling time of 0.1 sec. This needs to be improved in the future.

V. Conclusion

This paper proposes the model predictive PID control method that focuses on slip suppression for EV traction control. The proposed method uses MPC to perform control by calculating the optimum control input explicitly considering the constraints, using the calculation results to derive the realized PID gain in advance offline, creating a look up table, and referencing the suitable gain values in accordance with the conditions. In particular, constraints were also set for the variation of input torque ΔT_m in the low-speed range where the slip ratio λ fluctuates rapidly during start-off drive accompanying the minimal changes in wheel speed and vehicle body speed, and to satisfy this condition, the optimum PID gain is derived using an explicit-MPC framework based on the LQ criteria and a look up table is derived.

As an example, the results from a start-off (acceleration) performance simulation on an icy road are given. The simulation used MFC as the conventional method to perform a comparison verification with the proposed method. The results showed that when the proposed method is employed, the slip ratio is not only kept within $0.05 \leq \lambda \leq 0.25$, but that the speed response can be followed well for values in the vicinity of 0.1, which produces the optimum drive efficiency, allowing the drive force to be transmitted to the road surface without loss and improving the operational performance and vehicle behavior stability during start-off. In addition, although not discussed in this paper, employing the proposed method suppressed slip, which stabilized the yaw (the rotational motion of the vehicle as seen from directly above) meaning the proposed method is expected to allow the vehicle to move smoothly straight forward without unnecessary movement to the left or right.

This paper was limited to showing an example construction of the model predictive PID control system that can reduce the drive loss on icy roads using a simplified one wheel model, but to make the method practical, model predictive PID gain derivation for a variety of road conditions must be performed and a table created for more detailed two-wheel and four-wheel models. In addition, the suitability of the proposed method must be studied not only for the slip suppression addressed by this paper but also for overall driving including during braking. Even for this issue, however, the basic framework of the proposed method can be used as is and can also be expanded relatively easily to form a foundation for making practical EV high performance traction control systems and promoting further progress.

Acknowledgment

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