

Theoretical and Analytical Approaches for Investigating the Relations between Sediment Transport and Channel Shape

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Abstract—This study investigated the effect of cross sectional geometry on sediment transport rate. The processes of sediment transport are generally associated to environmental management, such as pollution caused by the forming of suspended sediment in the channel network of a watershed and preserving physical habitats and native vegetations, and engineering applications, such as the influence of sediment transport on hydraulic structures and flood control design. Many equations have been proposed for computing the sediment transport, the influence of many variables on sediment transport has been understood; however, the effect of other variables still requires further research. For open channel flow, sediment transport capacity is recognized to be a function of friction slope, flow velocity, grain size, grain roughness and form roughness, the hydraulic radius of the bed section and the type and quantity of vegetation cover. The effect of cross sectional geometry of the channel on sediment transport is one of the variables that need additional investigation. The width-depth ratio (W/d) is a comparative indicator of the channel shape. The width is the total distance across the channel and the depth is the mean depth of the channel. The mean depth is best calculated as total cross-sectional area divided by the top width. Channels with high W/d ratios tend to be shallow and wide, while channels with low (W/d) ratios tend to be narrow and deep. In this study, the effects of the width-depth ratio on sediment transport was demonstrated theoretically by inserting the shape factor in sediment continuity equation and analytically by utilizing the field data sets for Yalobusha River. It was found by utilizing the two approaches as a width-depth ratio increases the sediment transport decreases.

Keywords—Sediment transport, shape factor, hydraulic geometry, flow discharge, width depth ratio.

I. INTRODUCTION

THE physical structure of alluvial streams is a reflection of interactions among available energy, water, sediment and structural elements. Planform, cross-section, and profile are integrated features. Thus, altering one will affect the others, and alteration of any of these typically results in a change in the hydraulic and sediment transport characteristics of the channel. Modifications may include direct restoration (reconstruction of a channel) or incremental process restoration (installation of a structural feature to induce

change in a channel). The morphology of a stream is controlled by a dynamic balance between the amount of water flowing in the channel, the amount and size distribution of sediment delivered from upstream sources, the composition of the bed and banks, and the type and quantity of vegetation on the banks. When any of these components are altered, channel adjustments occur until a new dynamic equilibrium is achieved [1].

Alluvial river channels construct their own cross sections by transport and deposition of the unconsolidated sediment in which they are formed. Channel sizes and shapes reflect the quantities of water and sediment and the types of sediment imposed by their catchment hydrology. The influence of the sediment load introduced to a channel reach is illustrated by an inverse correlation between the channel width-depth ratio and the percentage of silt and clay in the perimeter sediment. The nature of the perimeter sediment depends on the dominant mode of sediment transport; a stream whose load is carried in suspension has a high percentage of silt and clay in its channel perimeter and a narrow, deep channel, whereas a bedload stream has a sandy perimeter and a wide, shallow cross section [2],[3].

Henderson's threshold channel equations [4] lead to that for a given discharge and slope, larger sediment load requires narrower channels, however, Bagnold's research [5], implies the contrasting in the results. In their researches the geometry of cross-section was not addressed. Generally streams do not incise unless the width/depth ratio is less than 10. This corresponds with higher shear stress and unit stream power [6].

The uncertainties associated with the variation of the physical characteristic of natural rivers, limited sample data, and inherent measurement errors will cause uncertainty in the parameters that describe the channel. To deal with those uncertainties that relate to the cross section the statistical characterization of channel cross section geometry will be used in this study. One way to describe irregular cross section geometry is by modeling flow depth as a power function of the channel geometry proprieties (top width, flow area and hydraulic radius) Gates et al. [7], [8]. Another way to deal with irregular cross section is by modeling the cross section as a function of effective width and effective width, then width: depth ratio can be found [9].

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The cross sectional form of natural channels is characteristically irregular in outline and locally variable. Width and depth give the gross dimensions of the channel but do not uniquely define cross section shape. Width–depth ratio is frequently used as index of the channel shape, even though it is not always the most appropriate [10].

The shape factor for irregular cross section can be calculated by using an effective depth, EFD, and the corresponding effective width, EFW as defined by USACE [9]

where,

$$EFD = \frac{\sum_{i=1}^{it} D_{avg} a_i D_{avg}^{2/3}}{\sum_{i=1}^{it} a_i D_{avg}^{2/3}} \quad (1)$$

$$EFW = \frac{\sum_{i=1}^{it} a_i D_{avg}^{2/3}}{EFD^{5/3}} \quad (2)$$

where: a_i =flow area of each trapezoidal element.

D_{avg} =average water depth of each trapezoidal element.

i_t =the total number of trapezoidal elements in a subsection.

The cross-section geometry is modeled through a set of parameters, Γ , that relate flow depth to flow area, A (m^2), and hydraulic radius, R (m). At a given station along the longitudinal (thalweg) axis of the channel, A and R are modeled as power functions of flow depth, h (m):

$$A = a_1 h^{a_2} + \varepsilon_A \quad (3)$$

and,

$$R = r_1 h^{r_2} + \varepsilon_R \quad (4)$$

$$\Gamma = \{a_1, a_2, r_1, r_2\} \quad (5)$$

The parameters a_2 and r_2 are indicators of cross-sectional shape (for example, for a rectangular cross section with vertical side slopes, $a_2 = 1$, and for a triangular cross section with uniform positive side slopes, $a_2 = 2$), while increasing values of a_1 and r_1 are indicators of increasing cross-sectional scale, or size, for a given shape. The terms ε_A (m^2) and ε_R (m) represent at-a-station random residuals that represent the uncertainty inherent (due to irregularity in the channel perimeter and due to survey errors) in predicting A and R , respectively, for a given flow depth [7].

II. SOURCE DATA

Data that are necessary to demonstrate the analytical approach in this research was obtained from existing field data of the Yalobusha River (natural channel), Mississippi. The HEC-RAS [11] model was used to calculate the backwater curve for the Yalobusha River; twenty cross sections were

entered in the model to show the correlation between cross section shape and sediment transport. The sediment transport will be determined by applying selected sediment transport equations.

III. PRESENT APPROACH

a) Theoretically by inserting the shape factor in sediment continuity equation and in some existing sediment transport formulae, the effect of shape factor on sediment transport can be assessed.

b) Analytically by including the existing cross-sections geometry for Yalobusha River in HEC-RAS model and calculating the backwater curve for this river. When the hydraulic parameters for every cross section are obtained, the sediment transport will be computed by using selected sediment transport equations. The possible relationship between shape factor and sediment transport can then be assessed.

A. Analytical Approach

The continuity equation [12] for sediment is:

$$(1 - \lambda) \frac{\rho_s}{\rho} \frac{\partial z}{\partial t} + \frac{\partial(Cuh)}{\partial x} + \frac{\partial Ch}{\partial t} = 0 \quad (6)$$

where

λ = the porosity of bed sediment

ρ_s = mass density of sediment

ρ = mass density of water

u = the mean flow velocity

h = the flow depth

C = the mean sediment concentration

z = the bed elevation

x = the distance along the channel

t = time

In the present research, the following effort was made to include a shape factor into the sediment continuity equation.

Multiplying equation (6) by the width of the cross section (W) yields

$$W[(1 - \lambda) \frac{\rho_s}{\rho} \frac{\partial z}{\partial t} + \frac{\partial(Cuh)}{\partial x} + \frac{\partial Ch}{\partial t}] = 0 \quad (7)$$

which can be expressed as:

$$(1 - \lambda) \frac{\rho_s}{\rho} \frac{\partial A_{bed}}{\partial t} + \frac{\partial(CuA_{c.s.})}{\partial x} + \frac{\partial CA_{c.s.}}{\partial t} = 0 \quad (8)$$

Term I

Term II

Term III

Term II=

$$Cu \frac{\partial A}{\partial x} + A \frac{\partial Cu}{\partial x} = Cu \frac{\partial A}{\partial x} + AC \frac{\partial u}{\partial x} + Au \frac{\partial C}{\partial x} \quad (9)$$

$$\text{let } A = a_1 h^{a_2} \quad (10)$$

where a_1 scale factor

a_2 shape factor

$$\Gamma = f(a_1, a_2)$$

where Γ is the statistical characterization of channel cross section geometry

$$\frac{\partial A}{\partial x} = \frac{dA}{dh} \left(\frac{\partial h}{\partial x} \right) + \frac{dA}{d\Gamma} \left(\frac{\partial \Gamma}{\partial x} \right)$$

$$\frac{dA}{dh} = a_1 a_2 h^{a_2-1}$$

$$\frac{dA}{d\Gamma} = \frac{dA}{da_1} + \frac{dA}{da_2} = h^{a_2} + a_1 h^{a_2} \ln h$$

$$\frac{\partial \Gamma}{\partial x} = \frac{\partial a_1}{\partial x} + \frac{\partial a_2}{\partial x} \approx \frac{\Delta a_1}{\Delta x} + \frac{\Delta a_2}{\Delta x}$$

$$\frac{\partial A}{\partial x} = \left[a_1 a_2 h^{a_2-1} \frac{\partial h}{\partial x} \right] + \left[h^{a_2} + a_1 h^{a_2} \ln h \right] \left(\frac{\partial a_1}{\partial x} + \frac{\partial a_2}{\partial x} \right)$$

$$\frac{\partial A}{\partial x} = \left(a_1 a_2 h^{a_2-1} \frac{\partial h}{\partial x} \right) + \left(h^{a_2} \frac{\partial a_1}{\partial x} + (a_1 h^{a_2} \ln h) \frac{\partial a_2}{\partial x} \right)$$

Term II

$$= Cu \left(a_1 a_2 h^{a_2-1} \frac{\partial h}{\partial x} + h^{a_2} \frac{\Delta a_1}{\Delta x} + (a_1 h^{a_2} \ln h) \frac{\Delta a_2}{\Delta x} \right) + a_1 h^{a_2} \left(C \frac{\partial u}{\partial x} + u \frac{\partial C}{\partial x} \right)$$

Term III

$$\frac{\partial CA_{c,s}}{\partial t} = C \frac{\partial A_{c,s}}{\partial t} + A \frac{\partial C}{\partial t}$$

$$\frac{\partial A}{\partial t} = \frac{dA}{dh} \left(\frac{\partial h}{\partial t} \right) + \frac{dA}{d\Gamma} \left(\frac{\partial \Gamma}{\partial t} \right)$$

From equations (10) and (11),

$$\frac{dA}{dh} = a_1 a_2 h^{a_2-1},$$

$$\frac{dA}{d\Gamma} = \frac{dA}{da_1} + \frac{dA}{da_2} = h^{a_2} + a_1 h^{a_2} \ln h$$

$$\frac{\partial \Gamma}{\partial t} = \frac{\partial a_1}{\partial t} + \frac{\partial a_2}{\partial t} \approx \frac{\Delta a_1}{\Delta t} + \frac{\Delta a_2}{\Delta t}$$

$$(11) \quad \frac{\partial A}{\partial t} = \left[\left(a_1 a_2 h^{a_2-1} \frac{\partial h}{\partial t} \right) \right] +$$

$$(12) \quad \left[\left(h^{a_2} + a_1 h^{a_2} \ln h \right) \left(\frac{\partial a_1}{\partial t} + \frac{\partial a_2}{\partial t} \right) \right]$$

$$(13) \quad \frac{\partial A}{\partial t} = \left(a_1 a_2 h^{a_2-1} \frac{\partial h}{\partial t} \right) + \left(h^{a_2} \frac{\partial a_1}{\partial t} + (a_1 h^{a_2} \ln h) \frac{\partial a_2}{\partial t} \right)$$

Term III

$$\frac{\partial CA}{\partial t} = C \left(a_1 a_2 h^{a_2-1} \frac{\partial h}{\partial t} + h^{a_2} \frac{\Delta a_1}{\Delta t} + a_1 h^{a_2} \ln h \frac{\Delta a_2}{\Delta t} \right) + a_1 h^{a_2} \left(\frac{\partial C}{\partial t} \right)$$

If we assume a steady supply of sediment then term III = 0

The continuity equation for sediment becomes:

$$\left((1-\lambda) \frac{\rho_s}{\rho} \frac{\partial A_{bed}}{\partial t} \right) + Cu \left(a_1 a_2 h^{a_2-1} \frac{\partial h}{\partial x} + h^{a_2} \frac{\Delta a_1}{\Delta x} + (a_1 h^{a_2} \ln h) \frac{\Delta a_2}{\Delta x} \right) + a_1 h^{a_2} \left(C \frac{\partial u}{\partial x} + u \frac{\partial C}{\partial x} \right) = 0$$

It is concluded from this equation that the shape factor a_2 and scale factor a_1 have a significant role in the sediment continuity equation.

B. Analytical Approach

The HEC-RAS computer model [13] was used to calculate surface water profile for steady, gradually-varied flow for the Yalobusha River. The basic computational procedure is based on the one dimensional energy equation, energy losses are evaluated by friction (Manning's equation), and the momentum equation is utilized in the situation where the water profile is rapidly varied. The steady flow component is capable of modeling subcritical, supercritical, and mixed flow regime water surface profiles.

Geometry of the river is represented by cross sections that are specified by coordinate points (stations and elevations) and the distances between cross sections. Twenty cross sections for Yalobusha River were used in the HEC-RAS model, flow data and boundary conditions were also entered to perform the calculations.

Once the backwater curve for this river was calculated, the hydraulic parameters for each cross section were obtained, and then the sediment transport was computed by Shen and Hung's transport equations [14] (Eq.26, and 27). See Tables I and II.

TABLE I
THE RESULTS FROM HEC-RAS MODEL

HEC-RAS Plan: Plan 05 River: Yalobusha Reach: Main Stem Profile: PF 1										
River Sta	Q Total (cfs)	Min Ch (ft)	Ch El (ft)	W.S. Elev (ft)	Crit W.S. (ft)	E.G. Elev (ft)	E.G. Slope (ft/ft)	Vel Chl (ft/s)	Flow Area (sq ft)	Top Width (ft)
20	3000	224.9	238.68	229.58	238.77	0.000133	2.38	1262.06	122.72	
19	3000	225.43	238.59	229.27	238.65	0.000091	1.97	1520.16	149	
18	3000	225.88	238.52	230.55	238.56	0.00007	1.57	1913.9	221.23	
17	3000	225.03	238.44	230.37	238.48	0.000076	1.7	1766.73	188.91	
16	3000	224.83	238.35	230.21	238.4	0.000098	1.81	1661.46	198.09	
15	3000	224.65	238.28	229.69	238.32	0.000068	1.64	1823.81	190.12	
14	3000	224.47	238.2	229.76	238.25	0.000081	1.76	1700.24	180.75	
13	3000	224.23	238.08	229.6	238.14	0.000107	1.89	1588.82	189.74	
12	3000	224.05	237.97	229.42	238.03	0.000136	2.03	1474.56	187.79	
11	3000	223.83	237.79	229.2	237.86	0.000167	2.15	1392.32	190.05	
10	3000	223.65	237.6	229.02	237.69	0.000207	2.33	1285.3	183.93	
9	3000	223.47	237.4	228.84	237.5	0.000211	2.51	1195.02	154.43	
8	3000	223.23	237.13	228.6	237.23	0.000235	2.57	1166.19	156.85	
7	3000	222.99	236.81	228.36	236.94	0.000241	2.92	1026.37	115.74	
6	3000	222.79	236.56	228.16	236.7	0.00026	2.93	1022.17	121.52	
5	3000	222.45	236.09	227.82	236.21	0.000301	2.84	1056	147.2	
4	3000	222.23	235.76	227.6	235.9	0.000264	3.02	993.1	114	
3	3000	222.03	235.5	227.4	235.64	0.000269	3.04	985.99	113.63	
2	3000	221.85	235.25	227.22	235.4	0.000275	3.07	978.21	113.2	
1	3000	221.63	234.94	227	235.09	0.000283	3.1	968.3	112.69	

TABLE II
THE RESULT FOR SEDIMENT TRANSPORT BY USING SHEN AND HUNG FORMULA AND (W/D) RATIO

w(ft/s) 0.36745407 Assume ds=1mm					
River Sta	Shen and Hung method			station (ft)	w/d
	sh	log Cppm	Cppm		
20	0.97106316	0.65997355	4.57060355	21029.79	8.50
19	0.96811018	-0.1590228	0.693389398	20029.8	10.68
18	0.96537658	-1.04541882	0.090070211	19029.81	19.11
17	0.96629358	-0.73346044	0.184730904	18029.82	15.43
16	0.96780267	-0.25231331	0.559353929	17029.83	15.38
15	0.96557252	-0.9774849	0.10532103	16129.84	14.84
14	0.96680916	-0.56464701	0.272491518	15229.85	14.02
13	0.96848154	-0.04844358	0.894450719	14029.86	13.76
12	0.96999787	0.3800927	2.399345016	13129.87	12.59
11	0.9712707	0.71254636	5.158772365	12029.88	11.57
10	0.97275195	1.0700096	11.74923522	11129.89	10.18
9	0.97337502	1.21140051	16.27048524	10229.9	9.21
8	0.97399735	1.34755196	22.26137357	9029.91	9.61
7	0.97503612	1.56394717	36.63930025	7829.92	8.32
6	0.97537846	1.63239044	42.89339635	6853.93	8.31
5	0.97576268	1.70756486	50.99937558	5129.95	8.65
4	0.97566373	1.68836985	48.79438517	4029.96	8.34
3	0.97579055	1.7129513	51.63584667	3029.97	8.35
2	0.97595475	1.74450486	55.52708259	2129.98	8.35
1	0.97614597	1.78086386	60.37593416	1029.99	8.36

Shen and Hung (1971) selected the bed material load concentration (C) as the dependent variable and the fall velocity (ω) in ft/sec of the median sediment particle of the bed sample, the flow velocity (V) in ft/sec, and energy slope (S) as independent variables. The concentration of bed sediment by weight (in ppm) is given as a power series of the flow parameter.

$$\log C_{ppm} = -107404.459 + 324214747 S_h - 326309.589 S_h^2 + 109503.872 S_h^3 \quad (26)$$

$$S_h = \left[\frac{VS^{0.57159}}{\omega^{0.31988}} \right]^{0.00750189} \quad (27)$$

where S_h = variable that depends on ω , V , and S .

Each cross section for Yalobusha River is defined by coordinates (X, Y), each cross section is divided into

subareas, and the area of each increment is computed below the water surface and between consecutive coordinates of the cross section as shown in Fig. 1. The equation for an incremental area, a_i , is:

$$a_i = \frac{(d_i + d_{i+1})W}{2} \quad (28)$$

where,

d_i, d_{i+1} = the left and right depth of each incremental area, respectively

W = width of an incremental area.

The equation for average depth for each trapezoidal element (D_{avg}) is:

$$D_{avg} = \frac{d_i + d_{i+1}}{2} \quad (29)$$

Once the values of a_i and D_{avg} are known, the shape factor (w/d) is calculated for each cross section by using equation (1) for effective depth and equation (2) for effective width.

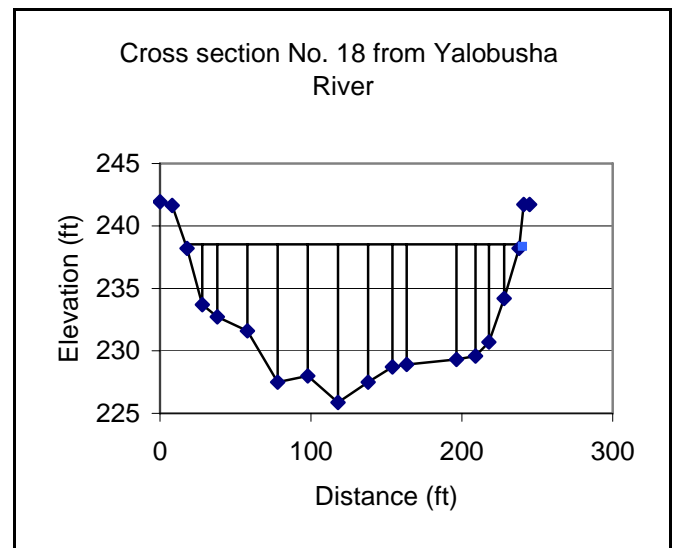


Fig. 1 An example shows the incremental areas

IV. CONCLUSIONS

Although great strides have been made in the knowledge of river mechanics, much research has been conducted to predict sediment transport; also many empirical and semi-empirical equations have been developed. There are many items, which require further research. For example none gives complete details about the effect of cross section geometry on sediment transport. The objective of this research is to investigate the relationship between cross section shape and sediment transport.

In this research the effect of cross section shape on sediment transport is demonstrated based on the Shen and Hung (1971) method.

The shape factor is inserted in the sediment continuity equation and the result is shown in equation (25). This equation can be used with the shape factor and scale factor for successive cross sections to predict the sediment concentration at the next cross section. This might be achieved by knowing the sediment concentration at the first cross section in addition to other hydraulic parameters that appear in that equation.

The effect of the shape factor on sediment transport is demonstrated by calculate the sediment transport for 20 cross sections in the Yalobosha River. There is strong relationship between sediment transport and the ratio between effective width to effective depth $R^2 = 0.87$ as shown in Fig. 2. It was found by utilizing the two approaches as a width-depth ratio increases the sediment transport decreases. The clarification of this consequence, an increasing in bank height generally results an increasing in shear stress and stream power and a potential for continued lowering of the streambed. The results of the increased slope and low width/depth ratio are to increase shear stress and stream power, causing incision and increasing in sediment transport. Any disruption in the natural energy balance and sediment transport will affect the morphological variables such as channel width/ depth ratio, cross section shape, pattern and profile that often lead to serious long-term adjustments such as aggradations and degradations.

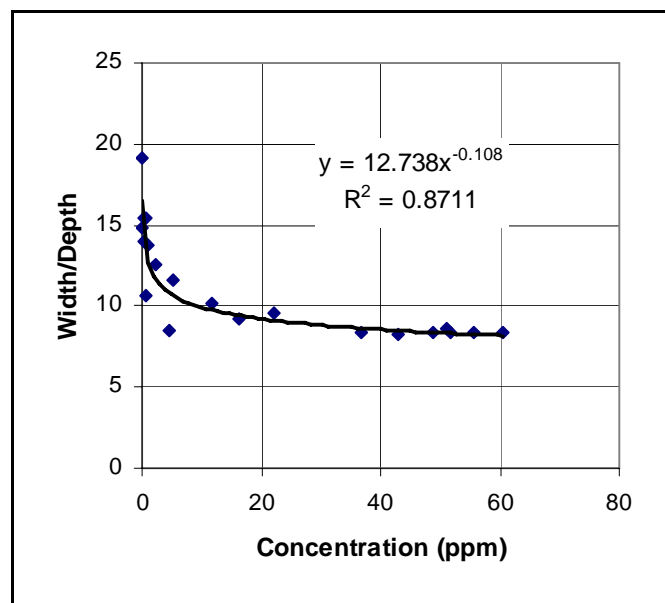


Fig. 2 The relation between shape factor (w/d) and sediment transport for Yalobosha River by using equation 26 (Shen and Hung, 1971)

List of Symbols

- a_1 = flow area of each trapezoidal element.
- a_1 = Cross section scale factor
- a_2 = Cross section shape factor
- A = cross-sectional area

- C = mean and total sediment concentration
- C_{ppm} = total sand and gravel concentration in parts per millions by weight
- d_i = local flow depth
- D_{av} = average water depth of each trapezoidal element.
- EFD = effective depth
- EFW = effective width
- h = flow depth
- i_t = the total number of trapezoidal elements in a subsection
- i = subscript for the appropriate data set
- R = hydraulic radius
- R^2 = coefficient of determination
- S = channel slope, energy slope, bed slope
- S_h = variable that depends on flow velocity, energy slope, and fall velocity
- t = time
- u = mean flow velocity
- V = flow velocity (m/s), depth-average velocity
- W = channel top width at the water surface
- W/d = width/depth ratio
- W/d = shape factor
- x = the distance along the channel
- z = the bed elevation
- ϵ_A, ϵ_R = represent at-a-station random residuals
- Γ = cross-section geometry parameter
- λ = the porosity of bed sediment
- ρ = density of water
- ρ_s = density of sediment
- σ = geometric bed material gradation coefficient
- ω = settling velocity

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