Sliding Mode Control based on Backstepping Approach for an UAV Type-Quadrotor

H. Bouadi, M. Bouchoucha, and M. Tadjine

Abstract—In this paper; we are interested principally in dynamic modelling of quadrotor while taking into account the high-order nonholonomic constraints in order to develop a new control scheme as well as the various physical phenomena, which can influence the dynamics of a flying structure. These permit us to introduce a new state-space representation. After, the use of Backstepping approach for the synthesis of tracking errors and Lyapunov functions, a sliding mode controller is developed in order to ensure Lyapunov stability, the handling of all system nonlinearities and desired tracking trajectories. Finally simulation results are also provided in order to illustrate the performances of the proposed controller.

Keywords—Dynamic modelling, nonholonomic constraints, Backstepping, Sliding mode.

I. INTRODUCTION

UNMANNED aerial vehicles (UAV) have shown a growing interest thanks to recent technological advancements, especially those related to instrumentation. They made possible the design of powerful systems (mini drones) endowed with real capacities of autonomous navigation at reasonable cost.

Despite the real progress made, researchers must still deal with serious difficulties, related to the control of such systems, particularly, in the presence of atmospheric turbulence. In addition, the navigation problem is complex and requires the perception of an often constrained and evolutionary environment, especially the case of low-altitude flights.

Nowadays, the mini-drones invade several application domains [4]: safety (monitoring of the airspace, urban and interurban traffic); natural risk management (monitoring of volcano activities); environmental protection (measurement of air pollution and forest monitoring); intervention in hostile sites (radioactive workspace and mine clearance), management of the large infrastructures (dams, high-tension lines and pipelines), agriculture and film production (aerial shooting).

In contrast to terrestrial mobile robots, for which it is often possible to limit the model to kinematics, the control of aerial robots (quadrotor) requires dynamics in order to account for gravity effects and aerodynamic forces [3].

In [7], authors propose a control-law based on the choice of a stabilizing Lyapunov function ensuring the desired tracking trajectories along $(X, Z)$ axis and roll angle. However, they do not take into account nonholonomic constraints. In [9], authors do not take into account frictions due to the aerodynamic torques nor drag forces or nonholonomic constraints. They propose a control-law based on backstepping in order to stabilize the complete system (i.e. translation and orientation). In [1], authors take into account the gyroscopic effects and show that the classical model-independent PD controller can stabilize asymptotically the attitude of the quadrotor aircraft. Moreover, they used a new Lyapunov function, which leads to an exponentially stabilizing controller based upon the PD$^2$ and the compensation of coriolis and gyroscopic torques. While in [2] the authors develop a PID controller in order to stabilize altitude.

Others papers; presented the sliding mode and high-order sliding mode respectively like an observer [14] and [15] in order to estimate the unmeasured states and the effects of the external disturbances such as wind and noise.

In this paper, based on the vectorial model form presented in [2] we are interested principally in the modelling of quadrotor to account for various parameters which affect the dynamics of a flying structure such as frictions due to the aerodynamic torques, drag forces along $(X, Y, Z)$ axis and gyroscopic effects which are identified in [2] for an experimental quadrotor and for high-order nonholonomic constraints [11]. Consequently, all these parameters supported the setting of the system under more complete and more realistic new state-space representation, which cannot be found easily in the literature being interested in the control laws synthesis for such systems.

Then, we present a control technique based on the development and the synthesis of a control algorithm based upon sliding mode based on backstepping approach ensuring the locally asymptotic stability and desired tracking trajectories expressed in term of the center of mass coordinates along $(X, Y, Z)$ axis and yaw angle, while the desired roll and pitch angles are deduced from nonholonomic constraints unlike to [9].

Finally all synthesized control laws are highlighted by simulations which gave results considered to be satisfactory.

H. Bouadi and M. Bouchoucha are with Control and Command Laboratory, EMP, BEB, 16111, Algiers, Algeria (e-mail: hakimusavj@yahoo.fr, mouloud_bouchoucha@yahoo.fr).

M. Tadjine is with Electrical Engineering Department, ENP, 10, Ave Hassen Badi, BP.182, EL-Harrah, Algiers, Algeria (e-mail: tadjine@yahoo.fr).
II. MODELLING

A. Quadrotor Dynamic Modelling

Fig. 1 Typical example of a quadrotor

The quadrotor have four propellers in cross configuration. The two pairs of propellers (1,3) and (2,4) as described in Fig. 2, turn in opposite directions. By varying the rotor speed, one can change the lift force and create motion. Thus, increasing or decreasing the four propeller’s speeds together generates vertical motion. Changing the 2 and 4 propeller’s speed conversely produces roll rotation coupled with lateral motion. Pitch rotation and the corresponding lateral motion; result from 1 and 3 propeller’s speed conversely modified. Yaw rotation is more subtle, as it results from the difference in the counter-torque between each pair of propellers.

Let \( E(O,X,Y,Z) \) denote an inertial frame, and \( B(o’,x’,y’,z’) \) denote a frame rigidly attached to the quadrotor as shown in Fig. 2.

\[ \Omega = \begin{bmatrix} 1 & 0 & -\sin \theta \\ 0 & \cos \phi \cos \theta & \sin \phi \sin \theta \\ 0 & -\sin \phi \cos \theta & \cos \phi \sin \theta \end{bmatrix} \]

\[ \Omega = \begin{bmatrix} I_x & 0 & 0 \\ 0 & I_y & 0 \\ 0 & 0 & I_z \end{bmatrix} \]

\[ \Omega = \begin{bmatrix} 0 & 0 & -\Omega_3 \\ 0 & 0 & \Omega_1 \\ -\Omega_2 & 0 & 0 \end{bmatrix} \]

% Equation 1

\[ \begin{align*}
\dot{\xi} &= \nu \\
m\ddot{\xi} &= F_f + F_i + F_g \\
\dot{R} &= RS(\Omega) \\
J\dot{\Omega} &= -\Omega \wedge J\Omega + \Gamma_f - \Gamma_a - \Gamma_g
\end{align*} \]

\( \xi \) is the position of the quadrotor center of mass with respect to the inertial frame. \( m \) is the total mass of the structure and \( J \in \mathbb{R}^{3 \times 3} \) is a symmetric positive definite constant inertia matrix of the quadrotor with respect to \( B \).

\[ F_f = \sum_{i=1}^{4} F_i \]

\[ F_i = K_p \omega_i^2 \]
Where $K_p$ is the lift coefficient and $\omega_i$ is the angular rotor speed.

$F_i$ is the resultant of the drag forces along ($X, Y, Z$) axis.

$$F_i = \begin{pmatrix} -K_{fix} & 0 & 0 \\ 0 & -K_{fay} & 0 \\ 0 & 0 & -K_{faz} \end{pmatrix} \dot{\xi}$$

(7)

Such as $K_{fix}, K_{fay}$ and $K_{faz}$ are the translation drag coefficients.

$F_g$ is the gravity force.

$$F_g = [0 \ 0 \ -mg]^T$$

(8)

$\Gamma_f$ is the moment developed by the quadrotor according to the body fixed frame. It is expressed as follows:

$$\Gamma_f = \begin{pmatrix} d(F_x - F'_x) \\ d(F_y - F'_y) \\ K_d(\alpha_x^2 - \alpha_z^2 + \alpha_z^2 - \alpha_x^2) \end{pmatrix}$$

(9)

d is the distance between the quadrotor center of mass and the rotation axis of propeller and $K_d$ is the drag coefficient.

$\Gamma_a$ is the resultant of aerodynamics frictions torques.

$$\Gamma_a = \begin{pmatrix} K_{fix} & 0 & 0 \\ 0 & K_{fay} & 0 \\ 0 & 0 & K_{faz} \end{pmatrix} \Omega^2$$

(10)

$K_{fix}, K_{fay}$ and $K_{faz}$ are the frictions aerodynamics coefficients.

$\Gamma_g$ is the resultant of torques due to the gyroscopic effects.

$$\Gamma_g = \sum_{i=1}^{4} \Omega_i \wedge J_i \begin{pmatrix} 0 \\ 0 \\ (\omega_i)^2 \end{pmatrix}$$

(11)

Such as $J_i$ is the rotor inertia.

Consequently the complete dynamic model which governs the quadrotor is as follows:

$$\begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \\ \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix} = \begin{bmatrix} \frac{1}{I_\phi} \left\{ \phi(\alpha_f - I_\phi \dot{\alpha}) - K_{ph} \dot{\phi} \right\} + J \bar{\alpha} \dot{\dot{U}}_1 \\ \frac{1}{I_\theta} \left\{ \theta(\alpha_f - I_\theta \dot{\alpha}) - K_{ph} \dot{\theta} \right\} + J \bar{\alpha} \dot{\dot{U}}_2 \\ \frac{1}{I_\psi} \left\{ \psi(\alpha_f - I_\psi \dot{\alpha}) - K_{ph} \dot{\psi} \right\} + J \bar{\alpha} \dot{\dot{U}}_3 \\ \frac{1}{m} \left\{ (C\dot{\alpha} + S\dot{\alpha}) \dot{U}_1 - K_{ph} \dot{x} \right\} \\ \frac{1}{m} \left\{ (C\dot{\alpha} + S\dot{\alpha}) \dot{U}_1 - K_{ph} \dot{y} \right\} \\ \frac{1}{m} \left\{ (C\dot{\alpha} + S\dot{\alpha}) \dot{U}_1 - K_{ph} \dot{z} \right\} \end{bmatrix}$$

(12)

With $U_1, U_2, U_3$ and $U_4$ are the control inputs of the system which are written according to the angular velocities of the four rotors as follows:

$$\begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \end{bmatrix} = \begin{bmatrix} K_p & K_p & K_p & K_p \\ -K_p & 0 & 0 & 0 \\ 0 & -K_p & 0 & K_p \\ K_d & -K_d & K_d & -K_d \end{bmatrix} \begin{bmatrix} \omega_x^2 \\ \omega_y^2 \\ \omega_z^2 \end{bmatrix}$$

(13)

and

$$\overline{\Omega} = (\omega_1 - \omega_2 + \omega_3 - \omega_4)$$

B. Nonholonomic Constraints

Taking into account nonholonomic constraints for our system is of major importance as are in compliance with physical laws and define the coupling between various states of the system.

From the equations of the translation dynamics (12) we can extract the expressions of the high-order nonholonomic constraints:

$$\begin{bmatrix} \tan \theta \\ \sin \phi \end{bmatrix} = \begin{bmatrix} \frac{\bar{y} - \frac{K_{ph}}{m} \dot{\alpha}}{\bar{z} + g - \frac{K_{ph}}{m} \dot{\alpha}} \cos \psi + \frac{\bar{y} - \frac{K_{ph}}{m} \dot{\alpha}}{\bar{z} + g - \frac{K_{ph}}{m} \dot{\alpha}} \sin \psi \\ -\left( \frac{\bar{z} - \frac{K_{ph}}{m} \dot{\alpha}}{m} \right) \sin \psi + \frac{\bar{z} - \frac{K_{ph}}{m} \dot{\alpha}}{m} \cos \psi \end{bmatrix}$$

(14)

C. Rotor Dynamic

The rotor is a unit constituted by D.C-motor actuating a propeller via a reducer. The D.C-motor is governed by the following dynamic equations:
\[ V = ri + L \frac{di}{dt} + k_m \omega \]
\[ k_m i = J_r \frac{d\omega}{dt} + C_s + k_r \omega^2 \]  

The different parameters of the motor are defined such:
- \( V \) : motor input.
- \( k_e, k_m \) : electrical and mechanical torque constant respectively.
- \( k_r \) : load constant torque.
- \( r \) : motor internal resistance.
- \( J_r \) : rotor inertia.
- \( C_s \) : solid friction.

Then, the model chosen for the rotor is as follows:
\[ \dot{\omega}_i = bV_i - \beta_0 - \beta_1 \omega_i - \beta_2 \omega_i^2 \]  

with:
\[ \beta_0 = \frac{C_s}{J_r}, \beta_1 = \frac{k_e}{rJ_r}, \beta_2 = \frac{k_r}{rJ_r} \text{ and } b = \frac{k_m}{rJ_r} \]

III. SLIDING MODE CONTROL OF THE QUADROTOR

The choice of this method is not fortuitous considering the major advantages it presents:
- It ensures Lyapunov stability.
- It ensures the robustness and all properties of the desired dynamics.
- It ensures the handling of all system nonlinearities.

The model (12) developed in the first part of this paper can be rewritten in the state-space form:
\[ \dot{X} = f(X) + g(X,U) + \delta \]  

\( X \) and \( X = [x_1, x_2, \ldots, x_n] \) is the state vector of the system such as:
\[ X = \begin{bmatrix} \phi, \psi, \theta, \dot{\phi}, \dot{\psi}, \dot{\theta}, x, y, z, \dot{x}, \dot{y}, \dot{z} \end{bmatrix}^T \]  

From (12) and (17) we obtain the following state representation:
\[ \begin{align*}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= a_1 x_4 x_6 + a_2 x_5^2 + a_3 \Omega x_4 + b U_2 \\
\dot{x}_3 &= x_4 \\
\dot{x}_4 &= a_2 x_7 x_8 + a_7 \Omega x_7 + b U_3 \\
\dot{x}_5 &= x_6 \\
\dot{x}_6 &= a_7 x_5 x_7 + a_8 x_6 + b U_4 \\
\dot{x}_7 &= x_8 \\
\dot{x}_8 &= a_8 x_6 + U_1 \frac{U_1}{m} \\
\dot{x}_9 &= x_{10} \\
\dot{x}_{10} &= a_{10} x_{10} + U_1 \frac{U_1}{m} \\
\dot{x}_{11} &= x_{12} \\
\dot{x}_{12} &= a_{11} x_{12} + C_s C x + U_1 - g \\
\end{align*} \]  

The state representation of the system under this form has never been developed before.

From high-order nonholonomic constraints developed in (14), roll (\( \phi \)) and pitch (\( \theta \)) angles depend not only on the yaw angle (\( \psi \)) but also on the movements along (\( X, Y, Z \)) axis and their dynamics. However the adopted control strategy is summarized in the control of two subsystems; the first relates to the position control while the second is that of the attitude control as shown it below the synoptic scheme:

Fig. 3 Synoptic scheme of the proposed controller

In this section we develop a sliding mode controller for the quadrotor based on Backstepping approach using the technique presented in [13].
Using the backstepping approach as a recursive algorithm for the control-laws synthesis, we simplify all the stages of calculation concerning the tracking errors and Lyapunov functions in the following way:

\[
z_i = \begin{cases} x_{id} - x_i & \text{if } i \in \{1, 3, 5, 7, 9, 11\} \\ x_i - x_{\varphi(i)} - \alpha_i z_i & \text{if } i \in \{2, 4, 6, 8, 10, 12\} \end{cases}
\]  
(21)

with \( \alpha_i > 0 \) \( \forall i \in [1, 12] \)

and:

\[
V_i = \begin{cases} \frac{1}{2} z_i^2 & \text{if } i \in \{1, 3, 5, 7, 9, 11\} \\ \frac{1}{2} (V_{i-1}^2 + z_i^2) & \text{if } i \in [2, 4, 6, 8, 10, 12] \end{cases}
\]  
(22)

The choice of the sliding surfaces is based upon the synthesized tracking errors which permitted us the synthesis of stabilizing control laws, so from (21) we define:

\[
S_x = z_2 = x_2 - \dot{x}_{id} - \alpha_i z_i \\
S_y = z_4 = x_4 - \dot{x}_{id} - \alpha_i z_3 \\
S_z = z_6 = x_6 - \dot{x}_{id} - \alpha_i z_5 \\
S_y = z_8 = x_8 - \dot{x}_{id} - \alpha_i z_7 \\
S_y = z_{10} = x_{10} - \dot{x}_{id} - \alpha_i z_9 \\
S_y = z_{12} = x_{12} - \dot{x}_{id} - \alpha_i z_{11}
\]  
(23)

Such as \( S_x, S_y, S_z, S_y, S_y, \) and \( S_z \) are the dynamic sliding surfaces.

To synthesize a stabilizing control law by sliding mode, the necessary sliding condition \( SS < 0 \) must be verified; so the synthesized stabilizing control laws are as follows:

\[
U = \frac{1}{h_l} \begin{bmatrix} -q_i \text{sign}(S_i) - k_i S_i - a_i x_i - a_i z_i - a_i \dot{\phi}_i + \alpha_i (\dot{\phi} - \dot{x}_i) \\ -q_i \text{sign}(S_i) - k_i S_i - a_i x_i - a_i z_i - a_i \dot{\phi}_i + \alpha_i (\dot{\phi} - \dot{x}_i) \\ -q_i \text{sign}(S_i) - k_i S_i - a_i x_i - a_i z_i - a_i \dot{\phi}_i + \alpha_i (\dot{\phi} - \dot{x}_i) \\ -q_i \text{sign}(S_i) - k_i S_i - a_i x_i - a_i z_i - a_i \dot{\phi}_i + \alpha_i (\dot{\phi} - \dot{x}_i) \\ -q_i \text{sign}(S_i) - k_i S_i - a_i x_i - a_i z_i - a_i \dot{\phi}_i + \alpha_i (\dot{\phi} - \dot{x}_i) \\ -q_i \text{sign}(S_i) - k_i S_i - a_i x_i - a_i z_i - a_i \dot{\phi}_i + \alpha_i (\dot{\phi} - \dot{x}_i) \end{bmatrix}
\]  
(24)

Such as \( (q_i, k_i) \in \mathbb{R}^{12} \).

**Proof**

We know a priori from (21) and (22) that:

\[
V_2 = \frac{1}{2} z_i^2 + \frac{1}{2} z_i^2 \\
z_2 = x_2 - \dot{x}_{id} - \alpha_i z_i
\]  
(25)

And from (23):

\[
S_x = z_2 = x_2 - \dot{x}_{id} - \alpha_i z_i
\]  
(26)

So:

\[
V_2 = \frac{1}{2} z_i^2 + \frac{1}{2} S_x^2
\]  
(27)

With:

\[
\dot{V}_2 = z_2 z_i + \dot{S}_x
\]  
(28)

The chosen law for the attractive surface is the time derivative of (26) satisfying \( \dot{S}_x < 0 \):

\[
\dot{S}_x = -q_i \text{sign}(S_i) - k_i S_i = \dot{x}_i - \dot{x}_{id} - \alpha_i \dot{z}_i
\]  
(29)

As for the Backstepping approach, the control input \( U_2 \) is extracted:

\[
U_2 = \frac{1}{h_l} \begin{bmatrix} -q_i \text{sign}(S_i) - k_i S_i - a_i x_i - a_i z_i - a_i \dot{\phi}_i + \alpha_i (\dot{\phi} - \dot{x}_i) \\ -q_i \text{sign}(S_i) - k_i S_i - a_i x_i - a_i z_i - a_i \dot{\phi}_i + \alpha_i (\dot{\phi} - \dot{x}_i) \\ -q_i \text{sign}(S_i) - k_i S_i - a_i x_i - a_i z_i - a_i \dot{\phi}_i + \alpha_i (\dot{\phi} - \dot{x}_i) \\ -q_i \text{sign}(S_i) - k_i S_i - a_i x_i - a_i z_i - a_i \dot{\phi}_i + \alpha_i (\dot{\phi} - \dot{x}_i) \\ -q_i \text{sign}(S_i) - k_i S_i - a_i x_i - a_i z_i - a_i \dot{\phi}_i + \alpha_i (\dot{\phi} - \dot{x}_i) \\ -q_i \text{sign}(S_i) - k_i S_i - a_i x_i - a_i z_i - a_i \dot{\phi}_i + \alpha_i (\dot{\phi} - \dot{x}_i) \end{bmatrix}
\]  
(30)

The same steps are followed to extract \( U_3, U_4, U_5, U_6, U_7, \) and \( U_8 \).

**IV. SIMULATION RESULTS**

The simulation results are obtained based on the following real parameters [2]:

\[
\begin{align*}
K_p &= 2.9842 \times 10^5 N.m/\text{rad}/s \\
K_d &= 2.3230 \times 10^7 N.m/\text{rad}/s \\
m &= 486g \\
d &= 25cm \\
J &= \text{diag}(3.8278; 3.8288; 7.6566) \times 10^{-4} N.m/\text{rad}/s^2 \\
K_p &= \text{diag}(5.5670; 5.5670; 6.3540) \times 10^{-4} N.m/\text{rad}/s \\
K_d &= \text{diag}(5.5670; 5.5670; 6.3540) \times 10^{-4} N.m/\text{rad}/s \\
J &= 2.8385 \times 10^{-4} N.m/\text{rad}/s^2 \\
\beta_x &= 189.63 \\
\beta_y &= 6.0612 \\
\beta_z &= 0.0122 \\
b &= 280.19
\end{align*}
\]

![Fig. 4 Global tracking trajectory of the quadrotor by sliding mode](image-url)
of mass coordinates of the system in spite of the complexity of the proposed model. As prospects we hope to develop other control techniques in order to improve the performances and to implement them on a real system.

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