Forced Vibration of a Fiber Metal Laminated Beam Containing a Delamination

Sh. Mirhosseini, Y. Haghighatfar, M. Sedighi

***Abstract*—**Forced vibration problem of a delaminated beam made of fiber metal laminates is studied in this paper. Firstly, a delamination is considered to divide the beam into four sections. The classic beam theory is assumed to dominate each section. The layers on two sides of the delamination are constrained to have the same deflection. This hypothesis approves the conditions of compatibility as well. Consequently, dynamic response of the beam is obtained by the means of differential transform method (DTM). In order to verify the correctness of the results, a model is constructed using commercial software ABAQUS 6.14. A linear spring with constant stiffness takes the effect of contact between delaminated layers into account. The attained semi-analytical outcomes are in great agreement with finite element analysis.

***Keywords*—**Delamination, forced vibration, finite element modelling, natural frequency.

I. Introduction

FIBER metal laminated beams are widely used in aerospace structures, and delamination is one of the most common types of damage in these components. It causes a significant stiffness reduction which makes the vibrational characteristics sensitive. It is of great importance to pinpoint the behavior of damaged structure in free and forced vibration as a health monitoring method. In order to investigate the delamination, Ramkumar et al. [1] presented a Timoshenko beam model which contains four beams neglecting the coupling between transverse and axial vibrations. They extracted natural frequencies and mode shapes of the problem. Wang et al. [2] expanded the model of Ramkumar et al. taking the aforementioned coupling into consideration. They used the assumptions of classic beam theory. Oveysi and Kharazi [3] studied buckling and post-buckling of composite laminate considering the contact between two delaminated layers. They modeled the contact as a linear spring. Anastasiadis and Simitses [4] also used a spring of constant stiffness to model the contact problem in buckling. Kargarnovin et al. [5] put forth a closed form solution to analyze dynamically the problem of forced vibration of a delaminated composite beam exposed to constant moving load. In this study, the delaminated beam is simulated with four connected subbeams.

In this study, forced dynamic problem of a delaminated beam made of fiber metal laminates is investigated. Natural frequencies of the beam can be simply derived using DTM. Then, with the assumption of constrained mode, delaminated layers are modeled. In this case, delaminated layers are presumed to have the same deflection which is physically feasible. With this hypothesis, the forced vibration problem would be analyzed through modal dynamics. Constrained mode problem turns out to give orthogonal mode shapes. In this section, the beam is divided in four sections for all of which Euler Bernoulli beam theory is utilized. Afterwards, dynamic response of the problem is validated through a finite element modelling in ABAQUS 6.14. Afterwards, the problem is investigated semi analytically and then verified making use of a simulation in ABAQUS 6.14. In the latter case, a spring of linear stiffness counts for the contact between delaminated layers.

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II. Governing Equations of the Problem

Fig. 1 shows a delaminated beam with four sections. In this paper, it is assumed that the delaminated parts (2 and 3) are constrained to move together which means that $W_2 = W_3$.

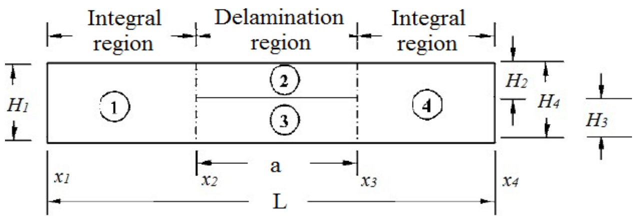

Fig. 1 Schematic of a delaminated beam

Euler Bernoulli beam equation of motion under transverse distributed load is defined as

$$\frac{\partial^2}{\partial x^2}\left[D\frac{\partial^2 W(x,t)}{\partial x^2}\right] + \rho A\frac{\partial^2 W(x,t)}{\partial t^2} = f(x,t) \quad (1)$$

in which $W(x,t)$ is the beam deflection, D is the bending stiffness, ρ is the density, A is the beam cross section, and $f(x,t)$ is the applied load. Taking advantage of modal analysis, a linear combination of beam normal modes is assumed as follows

$$W(x,t) = \sum_{j=1}^{\infty} w_j(x)\,\eta_j(t) \quad (2)$$

where $w_j(x)$ and $\eta_j(t)$ are normal modes and the mode shapes contribution in system response, respectively. Also j denotes modes number. Assuming harmonic response results in

$$\frac{d^2}{dx^2}\left[D\frac{d^2w_j(x)}{dx^2}\right]-\rho A\omega_j^2 w_j(x)=0 \tag{3}$$

In the above equation, ω_j designates j[th] mode frequency. It is possible to use (2) to write (1) as

$$\sum_{j=1}^{\infty}\frac{d^2}{dx^2}\left[D\frac{d^2w_j(x)}{dx^2}\right]\eta_j(t)+\rho A\sum_{j=1}^{\infty}w_j(x)\frac{d^2\eta_j(t)}{dt^2} = f(x,t) \tag{4}$$

Making use of (3), (4) can be expressed as

$$\rho A\sum_{j=1}^{\infty}\omega_j^2 w_j(x)\,\eta_j(t)+\rho A\sum_{j=1}^{\infty}w_j(x)\frac{d^2\eta_j(t)}{dt^2} = f(x,t) \tag{5}$$

Multiplying (5) with $w_k(x)$ and integrating on (0, L) domain leads to

$$\sum_{j=1}^{\infty}\eta_j(t)\int_0^L\rho A\,\omega_j^2 w_k(x)\,w_j(x)dx + \sum_{j=1}^{\infty}\frac{d^2\eta_j(t)}{dt^2}\int_0^L\rho A\,w_k(x)\,w_j(x)dx=\int_0^L w_k(x)\,f(x,t)dx \tag{6}$$

Constrained mode assumption for delamination results in orthogonality of mode shapes. Hence, we have the simplified equation for $k=j$

$$\frac{d^2\eta_j(t)}{dt^2}+\omega_{jj}^2\eta_j(t)=\frac{Q_j(t)}{m_{jj}}\ \ ,j=1,2,\ldots \tag{7}$$

In (7), $Q_j(t)$ is generalized force regarding j[th] mode defined as

$$Q_j(t)=\int_0^L w_j(x)\,f(x,t)dx\ \ ,j=1,2,\ldots \tag{8}$$

Generalized mass, m_{jj} is computed as

$$m_{jj}=\int_0^L\rho A(w_j(x))^2\,dx\ \ ,j=1,2,\ldots \tag{9}$$

Solution of (7) is demonstrated as

$$\eta_j(t)=A_j\cos(\omega_j t)+B_j\sin(\omega_j t) + \int_0^t Q_j\,(\tau)\sin\omega_j(t-\tau)\,d\tau \tag{10}$$

Substituting (10) in (2) gives the solution to (1)

$$W(x,t)=\sum_{j=1}^{\infty}[A_j\cos(\omega_j t)+B_j\sin(\omega_j t) + \frac{1}{\omega_j}\int_0^t Q_j\,(\tau)\sin\omega_j(t-\tau)\,d\tau]w_j(x) \tag{11}$$

The first two terms in above equation are related to free vibration that can be obtained using initial conditions. The last term shows forced vibration response of the system. In order to investigate a delaminated beam, natural frequencies are firstly derived. After computing natural frequencies of the beam by DTM method [6] and using step functions, mode shape of the whole beam can be expressed as

$$w_j(x)=w_1[j][u(x-x_1)-u(x-x_2)]+w_2[j][u(x-x_2)-u(x-x_3)]+w_4[j][u(x-x_3)-u(x-x_4)] \tag{12}$$

in which $w_1[j]$, $w_4[j]$ are continuous section's mode shapes and $w_2[j]$ is delaminated section's mode shape. It should be mentioned that $x_n(n=1,2,3,4)$ are boundary points at different sections and $u(x-x_0)$ is step function at x_0.

III. Finite Element Modelling of the Beam

In order to verify the results obtained, a finite element evaluation is performed in ABAQUS 6.14. In this regard, linear standard 8-noded solid elements (C3D8R) were used. The interaction in delaminated part is defined as a contact normal spring with the stiffness of $1.7\text{x}10^6$ N/m. Hexahedral linear standard elements were used in order to discretize the problem and satisfying the convergence. A two-step analysis is done in which firstly linear perturbation evaluation derives the natural frequencies of the beam. In the following step, modal dynamics, dynamic response of the beam exposed to external loads is attained.

IV. Results and Discussion

In this section, a load is applied to the beam and verified using finite element modelling. A cantilever beam of 150 mm length and 26.29 mm width is used. The specimen contains two aluminum layers of 0.44 mm thickness on top and bottom. The composite laminate in the middle of beam is made of glass epoxy with 1.52 total thickness and stacking sequence of $[0/90]_s$. Delamination between aluminum and composite layer, with 20.73 mm length, is located at a distance of 54.51 mm from clamped end. Mechanical properties of the beam components are tabulated in Table I.

TABLE I
Mechanical Properties of Aluminum and Glass Epoxy

Aluminum Grade 1000 properties						
ρ		E			ν	
2700 kg/m^3		72 GPa			0.32	
Composite properties						
ρ	E_1	E_2, E_3	G_{12}, G_{13}	G_{23}	ν_{12}, ν_{31}	ν_{32}
1540 kg/m^3	36 GPa	5GPa	2.7GPa	1.92GPa	0.25	0.301

A cosine applied force as $F=\cos(200t)$ is considered. The analytical and finite element dynamic response are shown in Figs. 2 and 3, respectively. As it is clear, the results are in great agreement. Additionally, FFT diagrams of the intact and delaminated beams using two solutions have been presented in Figs. 4-7. The first dominant frequency in all four following figures is the applied load frequency (200 rad/s or 31.74 Hz). It can be concluded that the beam starts to move at a frequency equal to the one of external load. Two remained frequencies belong to natural response of the problem. Comparing the two first natural frequencies shown in Figs. 4 and 5, it is understood that delamination results in reduction of beam stiffness leading to lower natural frequency; in these two figures, fundamental frequency of the intact beam (97.05 Hz) has reduced to 93.38 Hz. The second natural frequency (607.3 Hz) decreases to 561.5 Hz as well. The same conclusion holds for finite element solution in Figs. 6 and 7.

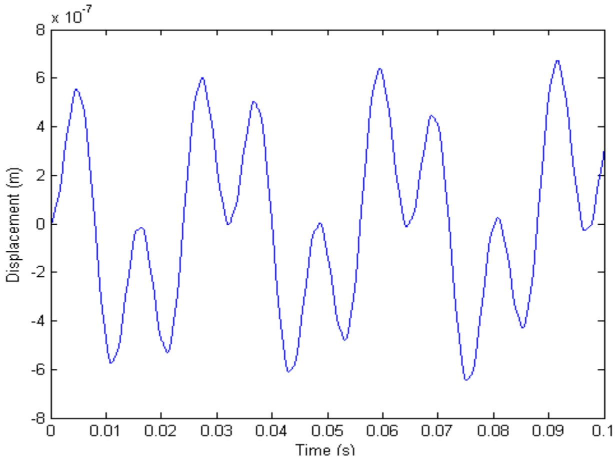

Fig. 2 Analytical solution of forced vibration problem exposed to cosine force

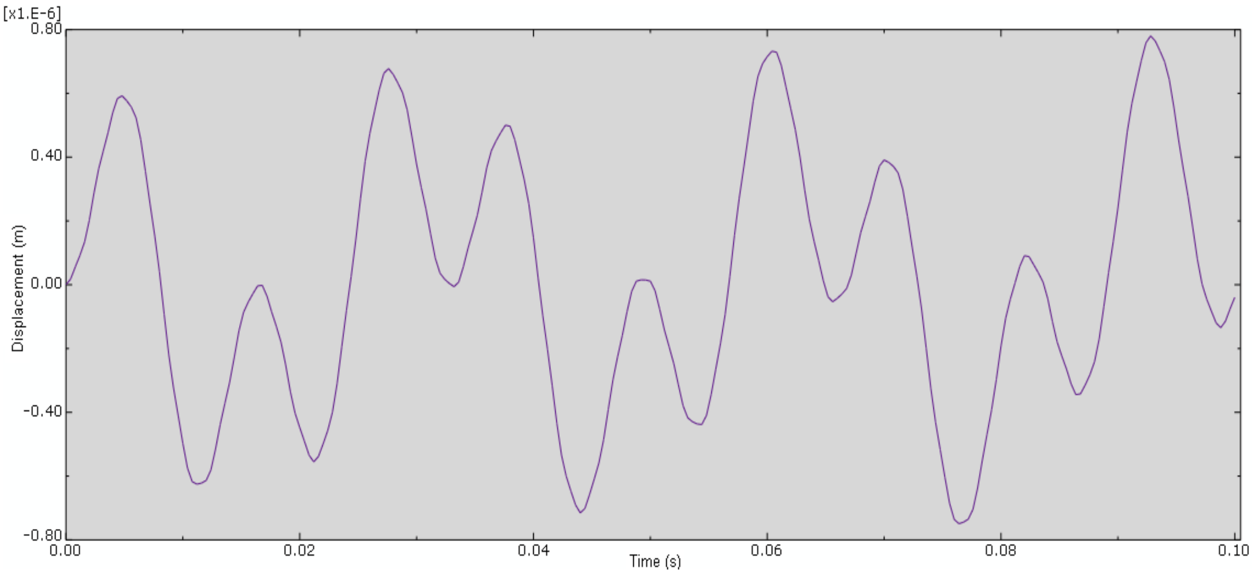

Fig. 3 Finite element solution of forced vibration problem exposed to cosine force

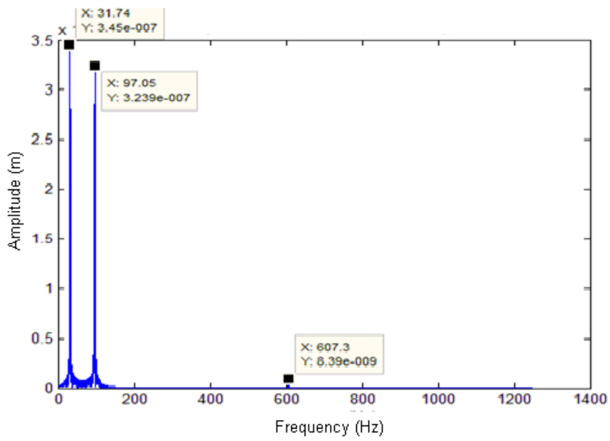

Fig. 4 FFT diagram of the intact beam using semi-analytical solution exposed to cosine force

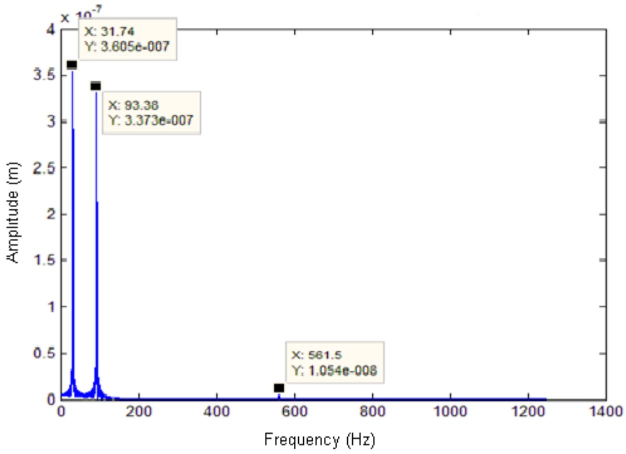

Fig. 5 FFT diagram of the delaminated beam using semi-analytical solution exposed to cosine force

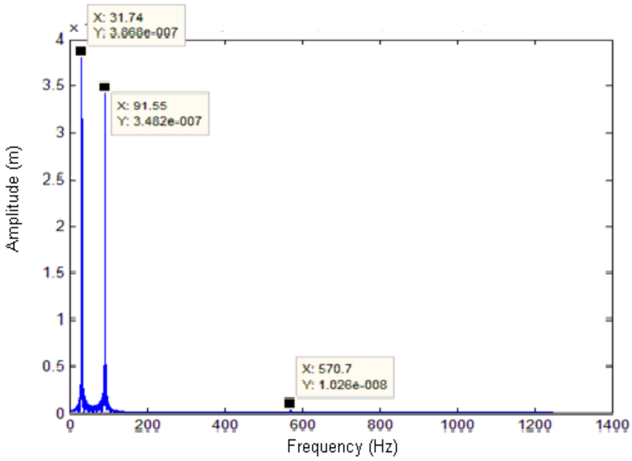

Fig. 6 FFT diagram of the intact beam using ABAQUS modelling exposed to cosine force

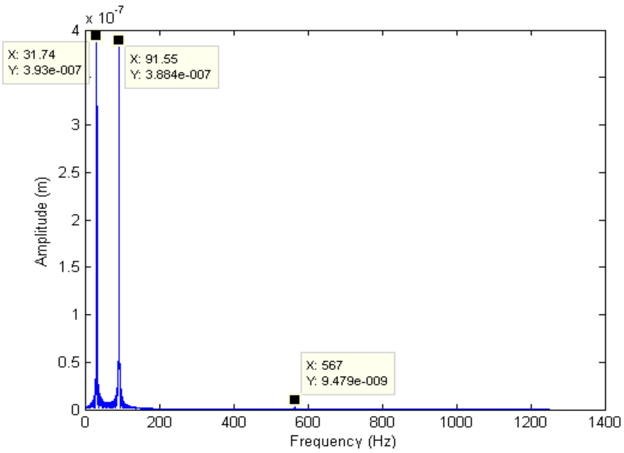

Fig. 7 FFT diagram of the delaminated beam using ABAQUS modelling exposed to cosine force

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