The Photon-Drag Effect in Cylindrical Quantum Wire with a Parabolic Potential

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Abstract—Using the quantum kinetic equation for electrons interacting with acoustic phonon, the density of the constant current associated with the drag of charge carriers in cylindrical quantum wire by a linearly polarized electromagnetic wave, a DC electric field and a laser radiation field is calculated. The density of the constant current is studied as a function of the frequency of electromagnetic wave, as well as the frequency of laser field and the basic elements of quantum wire with a parabolic potential. The analytic expression of the constant current density is numerically evaluated and plotted for a specific quantum wires GaAs/AlGaAs to show the dependence of the constant current density on above parameters. All these results of quantum wire compared with bulk semiconductors and superlattices to show the difference.

Keywords—Photon-drag effect, constant current density, quantum wire, parabolic potential.

I. INTRODUCTION

The photon-drag effect is explained by propagation electromagnetic wave carriers which absorb both energy and electromagnetic wave momentum, thereby electrons are generated with directed motion and a constant current is created in this direction. The presence of intense laser radiation can also influence electrical conductivity and kinetic effects in material [1]–[3]. The photon-drag effect has been researched in semiconductors [4]–[6], in superlattices [7]. In quantum wire, the photon drag effect is still open for study. In this paper, using the quantum kinetic equation for an electron system interacting with acoustic phonon is placed in a direct electric field, an electromagnetic wave, and the presence of an intense laser field in quantum wire with a parabolic potential, the constant current density of the photon-drag effect is calculated and numerical calculations are carried out with a specific GaAs/GaAsAl quantum wire.

II. CALCULATING THE CONSTANT CURRENT DESTINY OF THE PHOTON-DRAG EFFECT IN CYLINDRICAL QUANTUM WIRE WITH PARABOLIC POTENTIAL

We examine the electron system, which is placed in a linearly polarized electromagnetic wave \( \vec{E}(t) = \vec{E}_0 \cos(\omega t) \), \( \vec{H}(t) = [\vec{B}, \vec{E}(t)] \), in a DC electric field \( \vec{E}_0 \) and in a strong radiation field \( \vec{F}(t) = F_0 \sin(\Omega t) \). The Hamiltonian of the electron - phonon system in the quantum wire can be written as [8], [9] (using with \( \hbar = 1 \) unit and we suppose the axis 0z along the length of the wire):

\[
H = H_0 + U = \sum_{n,l,p} \varepsilon_{n,l,p} \hat{a}_{n,l,p}^\dagger \hat{a}_{n,l,p} + \sum_{q} \omega_q \hat{b}_q^\dagger \hat{b}_q +
\sum_{n,l,n',l'} C_{n,l,n',l'} \hat{a}_{n,l}^\dagger \hat{a}_{n',l'} + \hat{a}_{n,l}^\dagger \hat{a}_{n,l} (b_q + b_q^\dagger) \tag{1}
\]

where \( \hat{A}(t) \) is the vector potential of laser field (only the laser field affects the probability of scattering): \( \frac{1}{c} \hat{A}(t) = \vec{E}_0 \sin \Omega t \); \( \hat{a}_{n,l,p}^\dagger \) and \( \hat{a}_{n,l,p} \) (\( b_q^\dagger \) and \( b_q \)) are the creation and annihilation operators of electron (phonon); \( \vec{p} \) is the electron wave momentum along axis 0z; \( \vec{q} \) is phonon wave vector; \( \omega_q \) is the frequency of acoustic phonon; \( C_{n,l} \) is the electron-acoustic phonon interaction constant:

\[
C_{n,l} = \frac{\varepsilon_q}{2pV} V \rho \xi_n \xi_{n'} \xi_{l} \xi_{l'}; \quad \text{where } V, \rho, \xi_n \text{ and } \xi_{n'} \text{ are volume, the density, the acoustic velocity and the deformation potential constant, respectively; (n, l) and (n', l') are the quantum numbers of electron.}
\]

The electron energy takes the simple:

\[
\varepsilon_{n,l,p} = \frac{p^2}{2m} + \omega_q (2n + 1) \quad (n = 0, \pm 1, \pm 2, \ldots; \quad l = 1, 2, 3, \ldots)
\]

\[
I_{n,l,n',l'}(q) = \frac{2}{R^2} \int_{0}^{R} I_{n,l}(qR) \Psi_{n,1}(r) \Psi_{n',1}(r) dr
\]

where \( \Psi_{n,1}(r) = \frac{1}{J_{n+1}(A_n)} J_{n}(A_n) \frac{r}{R} \) is radial wave function, \( R \) is radius of wire, \( A_n \) is solution of the Bessel function of real argument \( J_{n}(A_n) = 0 \). This case particle system is set in a parabolic potential with quantum numbers \( n \) and form factor is not 1, this is the differences with the bulk semiconductor.

In order to establish the quantum kinetic equations for electrons in quantum wire, we use general quantum equations for the particle number operator or electron distribution function:

\[
i \frac{\partial \hat{f}_{n,l,p}^e(t)}{\partial t} = \langle \hat{a}_{n,l,p}^\dagger \hat{a}_{n,l,p}, H \rangle \tag{2}
\]
with $f_{n,l,p}(t) = a_{n,l,p}^+ a_{n,l,p} >_t$ is distribution function. From (1) and (2), we obtain the quantum kinetic equation for electrons in quantum wire (after supplement: A linearly polarized electromagnetic wave field and a direct electric field $E_0$):

$$\frac{\partial f_{n,l,p}(t)}{\partial t} = -\frac{e}{m} \sum_{n',l',p'} \frac{p_{n',l',p'}}{p_{n,l,p}} \delta (\varepsilon - \varepsilon_{n',l',p'}) + eE_0(t) + \omega_c \left[ \bar{p}_z \dot{\bar{h}}(t) \right]$$

for $n, l, p = 0, 1, 2, \ldots$.

We multiply both sides (3) by $z_n, p (e / m) p (\varepsilon)$ and obtain:

$$\frac{\partial f_{n,l,p}(t)}{\partial t} = -\frac{e}{m} \sum_{n',l',p'} \frac{p_{n',l',p'}}{p_{n,l,p}} \delta (\varepsilon - \varepsilon_{n',l',p'}) + eE_0(t) + \omega_c \left[ \bar{p}_z \dot{\bar{h}}(t) \right]$$

The constant current density in the form [10]:

$$\bar{j}_0 = \vec{R}_0(e) \text{d}e$$

with $\bar{j}_0 = -\frac{e}{m} \sum_{n,l,p} \bar{p}_z f_{n,l,p}(\bar{p}_z) \delta (\varepsilon - \varepsilon_{n,l,p})$ is partial current density.

For simplicity, we limit the problem to the case of $l = 0, \pm 1$. We multiply both sides (3) by $(-e / m) \bar{p}_z \delta (\varepsilon - \varepsilon_{n,l,p})$ are carry out the summation over $n, l$ and $\bar{p}_z$, we obtained:

$$\left( -io + \frac{1}{\tau(e)} \right) \bar{R}_0(e) = \bar{Q}_0(e) + \bar{S}_0(e) + \omega_c \left[ \bar{R}_0(e), \bar{h}(t) \right]$$

$$\left( -io + \frac{1}{\tau(e)} \right) \bar{R}_0^*(e) = \bar{Q}_0(e) + \bar{S}_0^*(e) + \omega_c \left[ \bar{R}_0(e), \bar{h}(t) \right]$$

where $\bar{h} = i \bar{H} / H$ is the unit vector of the magnetic field direction, $J_L = \frac{\bar{E}_0}{\gamma_0}$ is the Bessel function of real argument, $N_q$ is the time-independent component of distribution function of phonon: $N_q = \frac{1}{\exp(\beta \varepsilon_n) - 1}$.

The constant current density in the form [10]:

$$\bar{j}_0 = \frac{e}{m} \sum_{n,l,p} \bar{p}_z f_{n,l,p}(\bar{p}_z) \delta (\varepsilon - \varepsilon_{n,l,p})$$

The density of constant current:

$$\bar{R}_0(e) = \bar{R}_0(e) \bar{Q}_0(e) + \bar{S}_0(e) + \frac{2\alpha \varepsilon^2(\varepsilon)}{1 + \alpha^2 \varepsilon^2(\varepsilon)} \bar{Q}_0(e) \bar{R}_0(e) \text{Re} \left[ \frac{\bar{S}_0(e)}{1 - i \tau(e)} \right]$$

The density of constant current:

$$\bar{j}_0 = \int_0^\infty \bar{j}_0(e) \text{d}e = \frac{n_c e \frac{\bar{E}_0^2}{H} \bar{Q}_0^2(e)}{8\pi^2 \rho \Omega^2} \sum_{n,l,p} f_{n,l,p} \exp(-\beta \varepsilon_n) \left[ \frac{1}{1 + \alpha \varepsilon^2(\varepsilon)} \bar{R}_0(e) \bar{Q}_0(e) \text{Re} \left[ \frac{\bar{S}_0(e)}{1 - i \tau(e)} \right] \right]$$

with $\bar{j}_0 = \frac{e}{m} \sum_{n,l,p} \bar{p}_z f_{n,l,p}(\bar{p}_z) \delta (\varepsilon - \varepsilon_{n,l,p})$.
\[ A = \frac{n^2 \epsilon^2}{4\pi m^3 k_B T} \left( \frac{\hbar^2}{2m} \right)^{-2} \sum_{n,l} \exp\{-\beta \omega_0 (2n + 1 + 1)\} \]

\[ B = \sum_{n,l} \Gamma_{n,l} \exp\{-\beta \omega_0 (2n + 1 + 1)\} \]

\[ C = 12 \Psi \left( \frac{4,9}{2} \beta n_l^2 \right)_{\frac{2m}{\hbar^2}} + 20 \Psi \left( \frac{3,7}{2} \beta n_l^2 \right)_{\frac{2m}{\hbar^2}} + 2 \Psi \left( \frac{2,5}{2} \beta n_l^2 \right)_{\frac{2m}{\hbar^2}} + \]

\[ + 12 \Psi \left( \frac{4,9}{2} \beta n_l^2 \right)_{\frac{2m}{\hbar^2}} + 20 \Psi \left( \frac{3,7}{2} \beta n_l^2 \right)_{\frac{2m}{\hbar^2}} + 2 \Psi \left( \frac{2,5}{2} \beta n_l^2 \right)_{\frac{2m}{\hbar^2}} \]

\[ N_1 = -2m[2\omega_0(n - n') + \omega_0(1 - l') + \Omega] \]

\[ N_2 = -2m[2\omega_0(n - n') + \omega_0(1 - l') - \Omega] \]

\[
\Psi_{(a,b,x)} = \frac{1}{\Gamma(a)} \int_0^\infty e^{-x} x^{a-1} (1 + ax)^{b-1} dx
\]

is the Hypergeometric function.

Equation (14) shows the dependence of the constant current density on the intensity \( F \) and the frequency \( \Omega \) of the laser radiation field, the frequency \( \omega \) of the linearly polarized electromagnetic wave field, the frequency \( \omega_0 \) of the parabolic potential, the temperature \( T \) of the system. We also see the dependence of the constant current density on characteristic parameters for quantum wire such as: wave function; energy spectrum; form factor \( I_{n,l,n',l'} \) and potential barrier, that is the difference between the quantum wire, superlattices and bulk semiconductor. When \( \omega_0 \to 0 \) (where parabolic potential \( \Omega = \frac{m v^2}{2} \), \( m \) is effective mass of electron), the result will turn back to the photon drag effect in bulk semiconductors.

We will give a deeper insight into this dependence by carrying out a numerical assessment.

### III. NUMERICAL RESULTS AND DISCUSSION

In this section, we will survey, plot and discuss the expressions for \( j_{0z} \) for the case of a specific GaAs/GaAsAl quantum wire (we select: \( \epsilon \uparrow \uparrow 0x ; \hbar \uparrow \uparrow 0y \)):

\[
\begin{align*}
\dot{j}_{0z} &= \frac{n^2 \epsilon^2}{8\pi m^3 \nu C} \sum_{n,l,a} \Gamma_{n,l,a} \exp[-\beta \omega_0(2n+1+1)]N_{n,l}^2[C-A][E_0 + 2 \omega_0 \nu C]\left[ 1 - \frac{\omega_0 \nu C}{1 + \omega_0 \nu C} \right] \\
&= \frac{n^2 \epsilon^2}{8\pi m^3 \nu C} \sum_{n,l,a} \Gamma_{n,l,a} \exp[-\beta \omega_0(2n+1+1)]N_{n,l}^2[C-A][E_0 + 2 \omega_0 \nu C]\left[ 1 - \frac{\omega_0 \nu C}{1 + \omega_0 \nu C} \right]
\end{align*}
\]

(15)

The parameters used in the calculations are as follows [8]-[11]: \( m = 0.0665m_0 \) (the mass of free electron); \( \epsilon_0 = 50 \) meV; and \( \nu_0 \sim 10^{-11} s^{-1} \); \( \nu_0 = 5220m/s \); \( n_0 = 10^{21} m^{-3} \); \( \rho = 5.3 \times 10^4 kg/m^3 \); \( \xi = 2.2 \times 10^4 J \); \( \omega_0 = 5 \times 10^5 s^{-1} \); \( E_{d0} = 0.5 \times 10^6 (V/m) \); \( F = 1.2 \times 10^6 (N) \); \( \omega_0 = 2 \times 10^6 s^{-1} \); \( q = 2 \times 10^6 (kg.m/s) \).

**Fig. 1** The dependence of \( j_{0z} \) on the frequency of electromagnetic wave field with different values of \( \Omega \)

**Fig. 2** The dependence of \( j_{0z} \) on the frequency of radiation laser field with different values of \( T \)

The dependences of the constant current density on the frequency of the laser radiation field are shown in Fig. 2. We can see that the value of constant current density reduces nonlinearly when the frequency \( \Omega \) increases. And the more temperature \( T \) increases, the more the values of the current density rise.
This figure confirms once again that the constant current density along the oz axis also goes up.

The constant current density strongly depends on the frequency of the electromagnetic wave.

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In this paper, we have studied the drag effect in cylindrical quantum wire with a parabolic potential. In this case, one dimensional electron system is placed in a linearly polarized electromagnetic wave, a DC electric field and a laser radiation field at high frequency. We obtain the expressions for current density vector \( \mathbf{j}_0 \), in which, we plot and discuss the expressions for \( \mathbf{j}_{0z} \). And, the expressions of \( \mathbf{j}_{0z} \) show the dependence of \( \mathbf{j}_{0z} \) on the frequency \( \omega \) of the linearly polarized electromagnetic wave, on the temperature, the frequency \( \Omega \) of the intense laser radiation, and on the basic elements of quantum wire with a parabolic potential. The analytical results are numerically evaluated and plotted for a specific quantum wire GaAs/AlGaAs. From a comparison of the results of the quantum wire to semiconductors build \[11\], \[12\] and the superlattices \[8\] we see that: The basic differences between them is in wave function (with bulk semiconductor: electron gas is three-dimensional electron gas; with superlattices: electron gas is two-dimensional electron gas; with quantum well: electron gas is one-dimensional electron gas), in form factor \( I_{n,l,n',l'} \) (with bulk semiconductor: \( I_{n,l,n',l'} = 1 \)) and in potential barrier (with bulk semiconductor: \( V = 0 \)) that lead to the differences in expression and shape graphs of the constant current density.

REFERENCES


