Neuron Dynamics of Single-Compartment Traub Model for Hardware Implementations

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I. INTRODUCTION

The primary goal of conductance-based models is to incorporate as much cellular detail as is possible in order to have realistic neuron models. One of the most important features of these models is that they are biophysically compatible and hence neuroscientists, biologists, psychologists can, at certain level, study the properties and correlate directly parameters with their biological counterparts.

Besides the well known Hodgkin-Huxley (H-H) model; there are several conductance-based models which highlight different features in the neuron dynamics. One of the most used models, which take into account information about calcium ion channel Ca$^{2+}$, is Traub Model [1]; where the conductance $g_{Ca}$ is high-voltage activated type. Calcium dynamics is another important element in the chemical and electrical behavior in the neuron. This model can reproduce some burst patterns that H-H model does not exist [2].

The original work of Traub et al. consists in of a 19-compartment scheme for a CA3 hippocampal pyramidal neuron. However compute all these compartments to represent a single cell require big computational resources. In order to simplify this model, Pinsky and Rizel (P-R) [3] elaborated a 2-compartment model equivalent to original Traub, maintaining the essential ionic currents dynamics and capable of producing similar phenomena to the Traub model. The two-compartment model includes two parts: a soma-like, which has the Na$^{+}$ and K$^{+}$ activated currents; and a distal dendrite-like, where Ca$^{2+}$ activated and Ca$^{2+}$ dependent of potassium currents are considered.

The present research takes in consideration only the soma-like part of the model. Since our primary investigation is about biophysically compatible neuro-simulators for hardware implementations. It is important to keep good performance in order to have biological compatibility and reproduce results from real nervous systems.

In addition, it is of particular interest of this research to study the neuron dynamics behavior when two parameters change: applied current ($I_a$) and leakage conductance ($g_l$). Applied current is important since we want to have control about when neuron fires or produce bursting/periodic spikes. This is useful for some kind of experiments such as tuning parameters in learning process, neuron excitability tests, measure bursting properties of the neuron, etc.[4]-[6].

Bifurcations mechanisms involved are involved in the generation of action potentials (spikes) by neurons. Hence a bifurcation study can determined the neuro-computational properties and dynamics of the cells [7], [8].

Using common parameter values of Traub model at resting state, it produces periodic spikes, even without any current applied; then it is fundamental to maintain neuron at equilibrium voltage when neuron it is at resting state and without compromising the dynamics of the neuron. This can be done adjusting the value of leakage conductance. Another interesting aspect about leakage conductance, which mainly consists of chloride ions, is that it is related with some ion-channel diseases and can be used in feedback control schemes to lead new electrical stimulation systems [5]-[9]. This issue will be topic of discussion for future work.

We intend that our neuro-simulator platform [10] can simulate either single or two-compartment (soma+dendrite) type of neurons. Here we study the single compartment neuron dynamics through bifurcation techniques in order to analyze the excitability properties when a particular current is injected to the neuron.

II. MATHEMATICS OF TRAUB MODEL

Equation (1) represent the soma-like based on the original work of Pinsky-Rinzel [3]. Where $C_m$ is the capacitance for the cell membrane ($\mu F/cm^2$); $V$ represent the membrane potential (mV)$q_{ij}$ and $E_i$ are the maximal conductance and the equilibrium potential of the ionic specie $i$ respectively (mS/cm$^2$ and mV); $I_a$ stands for injected current ($\mu A/cm^2$); $m$, $h$ and $n$ are the unitless activation/inactivation gating variables having the form of the first kinetic formula. These variables
depends of the functions $\alpha(V)$ and $\beta(V)$, which describe the transition rates between open and closed states of the channels.

Like other conductance-based models, the gating variables for Traub Model have specific representations for $\alpha(V)$ and $\beta(V)$ measured experimentally, Table I shows these values for a resting potential of $-60 \text{ mV}$.

$$V' = \left[-g_{Na}m^2h(V-E_{Na}) - g_{KDR}n(V-E_{KDR}) - g_L(V-E_L) + I_e \right]/C_m$$

$$m' = \alpha_m(V)(1-m) - \beta_m(V)m$$

$$n' = \alpha_n(V)(1-n) - \beta_n(V)n$$

$$h' = \alpha_h(V)(1-h) - \beta_h(V)h$$

(1)

In order to analyze the stability for this equilibrium points and since the Jacobian matrix $J(X_e)$ at the equilibrium point $X_e$ is used in the linearization of original non-linear system; the Jacobian matrix is calculated, where $f_v, f_m, f_h,$ and $f_n$ have the form of the right side in (1).

$$J = \begin{bmatrix}
    \frac{df_v}{dV} & \frac{df_v}{dm} & \frac{df_v}{dh} \\
    \frac{df_m}{dV} & \frac{df_m}{dm} & \frac{df_m}{dh} \\
    \frac{df_h}{dV} & \frac{df_h}{dm} & \frac{df_h}{dh} \\
    \frac{df_n}{dV} & \frac{df_n}{dm} & \frac{df_n}{dh}
\end{bmatrix}$$

The Eigen-values of equilibrium points A and B are solved and interpret next:

$$\Psi_A = -10.6314; -0.0132; -0.4818; -0.3078$$

$$\Psi_B = -3.075 + 7.206i; -3.075 - 7.206i; 1.427; -0.291$$

The equilibrium point A, with its four negative values is asymptotically stable node; i.e. membrane potential will remain at its rest state unless some excitation pulse generate an action potential, then membrane voltage will return to the equilibrium state again. According to point B, because the presence of positive value it is unstable, moreover, since it has a couple of conjugate eigen-values, which makes the appearance of Hopf bifurcation, i.e. stable or unstable limit cycles [11], [12]. As we now, limit cycles is the graphical representation of bursting and periodic spiking for a neuron.

So, if we want to study the effects when parameters such $I_e$ or $g_L$ change, then we need to focus on point B. So equilibrium and initial condition in point B is used to start the bifurcation analysis using the tool XPPAUT.
There are three main points it is worth to focus. First, P1 (V≈-58.64, Ie≈0) is in fact, the equilibrium point A and it is also a bifurcation point, if voltage and current becomes more negative, the membrane voltage tends to go a specific stable fixed point and gets higher (more negative) as current is more negative. However, if membrane voltage slightly increases (becomes less negative) then it goes to an unstable fixed points zone, and it start to oscillate. This is not good, especially if we want that neuron stay at its resting value when 0 mA current is applied, instead of start oscillating. This will be fixed in Section IV.

P2 (V≈-28.6, Ie≈90) is where a Hopf bifurcation appears and stable periodic orbits increase in amplitude as current tends to zero, i.e. when current starts to increase from zero the neuron start to fires periodically with spikes amplitudes near of 15 mV, whereas current keeps increasing, the neuron maintains firing periodically but with less amplitude and more frequency, eventually when current applied reach 90 mA, then membrane voltage tends to a fixed stable as we can see the right red line after P2. However is the value of 65 mA where the membrane output can still consider as action potentials, i.e. spikes amplitude is bigger than -20 mV threshold.

Regarding leakage conductance parameter, a bifurcation diagram is shown in Fig. 4 when applied current is zero. The important point to notice is P1, where there is a bifurcation point at $g_l = 0.4522\text{mS/cm}^2$, values of leakage conductance smaller than this value makes the neuron unstable, producing action potentials even if the neuron has no stimulus. If this kind of model is intended to be used in neural networks with learning process, this is not a desired behavior. So it is necessary to choose the proper value of leakage conductance where we can guarantee the neuron will stay at stable value when there is no current applied, this is done by choosing a value bigger than 0.4522 mS/cm$^2$, where as we can see in Fig. 4, the neuron will remain at resting value of 60 mV approximately (red line). The equilibrium point A was achieved with a leakage conductance of 0.5 mS/cm$^2$.

In point P2, when $g_l = -5.8$ a Hopf bifurcation rises with unstable orbits (blue spots); nevertheless it is a mathematical bifurcation point, from a biophysical point of view does not have relevance due to negative values for leakage conductance.

The complete list of parameters used for this study is:

- $g_{Na} = 30 \text{ mS/cm}^2$; $g_K = 15 \text{ mS/cm}^2$; $g_l = 0.5 \text{ mS/cm}^2$;
- $E_{Na} = 40 \text{ mV}$; $E_K = -75 \text{ mV}$; $E_l = -60 \text{ mV}$; $C_m = 3 \mu\text{F/cm}^2$. These values are consistent with the original work in [1]-[3].

IV. HARDWARE IMPLEMENTATION TEST

Finally the single compartment Traub model was tested in a FPGA device. Following a scheme of state machines, floating-point arithmetic units and BRAMs for store internal results, a soma-like neuroprocessor was developed. This implementation is part of the project “Efficient and biophysical accurate neuroprocessors”. Because this neuroprocessors are implemented in digital programmable devices, (1) need to be solved by numerical methods. Exponential Euler method offers good tradeoff between system stability and computational resources for implementation. Because the development of such neuroprocessors is not the principal discussion on this paper, more details about this platform can be found in [2]-[10].
Several tests were run on this platform and Fig. 5 presents the results for three different applied currents. The frequency of spikes is consistent with results obtained by current bifurcation. And when a current bigger than 90 mA is injected; a short transient response occurs and then membrane voltage goes to a steady state of 28 mV.

![Figure 5](image-url)

**Fig. 5** Periodic spikes hardware implementation results for different applied current. a) 10 mA, b) 65 mA and c) 95 mA

V. CONCLUSIONS AND FUTURE WORK

A mathematical analysis for neuron dynamics was developed. The Traub model used is one of the most important conductance-based models and a single compartment representation was analyzed. The leakage conductance value was tuned in order the neuron remains at fixed value when it is at resting state. This parameter is the best option to change if does not want to compromise the dynamic of the original model.

Through bifurcation analysis, it was detected the stable and unstable solutions for this model. A Hopf bifurcation was discovered at the point \( I = 90 \text{ mA} \), given to the current range \([0, 90] \text{ mA}\) a set of stable periodic orbits with different action potentials amplitudes. The frequencies range for this periodic orbits are from 50 to 341 Hz. All this information is relevant since it is the first analysis done for a single compartment Traub model and it is well established quantitative current values where we can have knowledge of what kind of response from the neuron we can expect; this is particularly important if we want to use this neuro processors in a neural network and incorporate a learning process; it is necessary to know which values of current will make the neuron fires.

As future work we are planning to make a similar analysis for the complete two-compartment Traub model, taking in consideration the dendrite calcium ion channel information. Also using the same bifurcation analysis found relevant information related with ion-channel diseases in order to implement neural control models which helps in this particular issues.

REFERENCES


