Unsteady Flow and Heat Transfer of Nanofluid from Circular Tube in Cross-Flow

H. Bayat, M. Majidi, M. Bolhasani, A. Karbalaie Alilou, A. Mirabdolah Lavasani

Abstract—Unsteady flow and heat transfer from a circular cylinder in cross-flow is studied numerically. The governing equations are solved by using finite volume method. Reynolds number varies in range of 50 to 200; in this range flow is considered to be laminar and unsteady. Al$_2$O$_3$ nanoparticle with volume fraction in range of 5% to 20% is added to pure water. Effects of adding nanoparticle to pure water on lift and drag coefficient and Nusselt number is presented. Addition of Al$_2$O$_3$ has inconsiderable effect on the value of drags and lift coefficient. However, it has significant effect on heat transfer; results show that heat transfer of Al$_2$O$_3$ nanofluid is about 9% to 36% higher than pure water.

Keywords—Nanofluid, heat transfer, unsteady flow, forced convection, cross-flow.

I. INTRODUCTION

Flow and heat transfer from bluff bodies have many industrial application, such as HVAC system, heat exchangers and so on. Recently, nanofluids are used for increasing heat transfer rate inside circular and non-circular channel. Xuan and Li [1] studied effect of using nanofluids in increasing heat transfer. They found that nanofluids have great potential in enhancing the heat transfer process.

Hussein et al. [2] studied effect of cross sectional area of tube on friction factor and heat transfer of nanofluid. They used TiO$_2$ nanoparticle with volume fraction 1%-2.5% which was suspended in water as base fluid. They found that by increasing volume concentration of nanofluid friction factor and heat transfer increases. Moreover, they concluded that a flat tube has the highest value of heat transfer coefficient followed by elliptical and circular tube.

Tsai and Chein [3] analytically studied performance of microchannel heat sink (MCHS) by using copper-water and carbon nanotube-water nanofluid as coolants. Their results show that presence of nanoparticle reduced the temperature difference between MCHS bottom wall and bulk nanofluid compared to pure fluid. Non-circular cylinder [4]-[10] have lower pressure drop; as a result its thermal-hydraulic performance is higher than circular tube.

Wen and Ding [11] experimentally investigated convective heat transfer of nanofluid, made of γ-Al2O3 nanoparticle and de-ionized water in laminar flow. Their result showed considerable enhancement in heat transfer, particularly in the entrance region.

Heidary and Kermani [12] studied heat transfer enhancement of nanofluid inside a channel with blocks attached to the bottom wall. They found that by using nanoparticle and usage of block on the hot wall would lead to enhancement in heat transfer.

Hamad [13] analytically studied natural convective flow and heat transfer of an incompressible nanofluid pas a semi-infinite vertical stretching sheet in the presence of a magnetic field. He found that by increasing of the magnetic parameter, the momentum boundary layer thickness decreases, while the thermal boundary layer thickness increases.

Noie et al. [14] used Al2O3 nanoparticle to enhance heat transfer in a two-phase closed thermosyphon (TPCT). Their experimental results showed that for different input powers, when they used Al2O3/Water instead of pure water, the efficiency of the TPCT increased up to 14.7%.

Ghazvini and Shokouhmand [15] analytically and numerically studied effect of using CuO/water nanofluids as coolant of micro-channel heat sink. They used two common analytical approaches: the Fin model and the porous media approach. In both approaches, using nanofluid enhances heat transfer. Mansour et al. [16] simulated mixed convection in a square lid-driven cavity which is filled with water-based nanofluid partially heat from below by using finite difference method. They found that by increasing solid volume fraction, average Nusselt number increases. Lavasani and Bayat [17]-[20] studied flow and heat transfer from two cam-shaped tube in cross-flow of air with different arrangement. Mirabdolah Lavasani et al. [21] and Bayat et al. [22] studied experimentally pressure drop and heat transfer from cam-shaped tube in cross-flow of air. Their results showed that cam-shaped tube bank compare to circular bank performs better.

Ahmad and Pop [23] numerically studied mixed convection boundary layer flow past a vertical flat plate. Khan and Aziz [24] numerically investigated natural convective of nanofluid over a vertical plate with constant heat flux. Their results show that for a fixed Lewis number as the Prandtl number increases the dimensionless skin friction and Nusselt number increased. Shafahi et al. [25] used analytical method to investigate effect of using nanofluid on thermal performance of rectangular and disk-shaped heat pipes. They found that using nanofluid will lead to reduction of the thermal resistance of the flat-shaped heat pipe and thermal performance enhanced compared to regular fluid.

Ghasemi and Aminossadati [26] numerically investigated
mixed convection in a lid-driven triangular enclosure filled with water-Al2O3. Their results show that addition of nanofluid improves the heat transfer rate of the enclosure. Kathiravan et al. [27] conducted an experiment to investigate pool boiling characteristics of copper nanofluid over a flat plate. They found that critical heat flux of copper-water nanofluid is higher than pure water. Soltani et al. [28] experimentally studied non-Newtonian nanofluids pool boiling characteristics. They found that adding nanoparticle to non-Newtonian improved boiling heat transfer performance.

Because there is lack of study in behavior of nanofluid around cylinder and due its large application, heat transfer from a circular cylinder with constant wall temperature is investigated by using numerical method.

II. Problem Description

A. Governing Equations

Al2O3 nanoparticle with volume fraction of 5% to 20% added to pure water and it passes from a circular tube with diameter of 2 cm with constant temperature of 400 K. The typical solution domain and the cylinder boundary definition and nomenclature used in this work are shown in Fig. 1. The inlet flow has a uniform velocity \( U_{\infty} \). The velocity range considered only covers laminar flow conditions. The solution domain is bounded by the inlet, the outlet, and by the plane confining walls, AB and CD. These are treated as solid walls, while AC and BD are the flow inlet and outlet planes.

In order to decrease the effect of entrance and outlet regions, the upstream and downstream lengths are 15\( D_{eq} \) and 50\( D_{eq} \), respectively and for neglecting the wall effects on cylinders the distance between walls is 30\( D_{eq} \).

Equations (1)-(4) are written for conservation of mass, momentum and energy in two dimensions. Cartesian velocity components \( U \) and \( V \) are used, and it has been assumed that the flow is steady and laminar, while the fluid is incompressible and Newtonian with constant thermal and transport properties. Furthermore, the effects of buoyancy and viscous dissipation are neglected.

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}
\]

\[
\frac{\partial u}{\partial t} + U \frac{\partial u}{\partial x} + V \frac{\partial u}{\partial y} = -\frac{1}{\rho_{nf}} \frac{\partial p}{\partial x} + \nu_{nf} \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) \tag{2}
\]

\[
\frac{\partial v}{\partial t} + U \frac{\partial v}{\partial x} + V \frac{\partial v}{\partial y} = -\frac{1}{\rho_{nf}} \frac{\partial p}{\partial y} + \nu_{nf} \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) \tag{3}
\]

\[
\frac{\partial T}{\partial t} + U \frac{\partial T}{\partial x} + V \frac{\partial T}{\partial y} = \alpha_{nf} \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) \tag{4}
\]

where,

\[
\rho_{nf} = \frac{\mu_{nf}}{k_{nf}} \tag{5}
\]

\[
\alpha_{nf} = \frac{k_{nf}}{(\rho C_p)_{nf}} \tag{6}
\]

Nanoparticles are considered to be well dispersed within the base fluid. By introducing the nanofluid volume fraction \( \phi \), density and heat capacity of nanofluid is calculated from the classical models [29]:

\[
\rho_{nf} = (1-\phi)\rho_f + \phi \rho_p \tag{7}
\]

\[
(\rho C_p)_{nf} = (1-\phi)(\rho C_p)_f + \phi(\rho C_p)_p \tag{8}
\]

where \( C_p \) and \( \rho \) are respectively heat capacity and density. Index \( f \), \( p \) and \( nf \) refer to base fluid, nanoparticle and nanofluid, respectively.

Equation (9) is suggested by [30] for calculating the dynamic viscosity of the nanofluid:

\[
\mu_{nf} = \frac{\mu_f}{(1-\phi)^2} \tag{9}
\]

The effective thermal conductivity of fluid is calculated from Maxwell-Garnett’s model.

\[
k_{nf} = \frac{k_f - k_p + 2k_f - 2\phi(k_f - k_p)}{k_f + 2k_p + \phi(k_f - k_p)} \tag{10}
\]

The properties of pure water and Al2O3 are presented in Table I.

Drag and lift coefficients are defined as:

\[
C_D = \frac{F_D}{0.5 \rho U_{\infty}^2 D} \tag{11}
\]

\[
C_L = \frac{F_L}{0.5 \rho U_{\infty}^2 D} \tag{12}
\]

Nusselt number is defined with:

\[
Nu = \frac{qD}{Ak \Delta T} \tag{13}
\]
where \( q \) is the total rate of heat transfer to the fluid and \( A \) is the total surface area of tubes. The temperature of the cylinders wall is 350 K, the bulk temperature of the cross-flow nanofluid is 300 K, and \( \Delta T \) is the difference between these temperatures.

### TABLE I

<table>
<thead>
<tr>
<th>Properties of Water and Alumina</th>
<th>( \rho ) (kg/m(^3))</th>
<th>( C_p ) (J/kgK)</th>
<th>( k ) (W/mK)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pure water</td>
<td>997.1</td>
<td>4179</td>
<td>0.613</td>
</tr>
<tr>
<td>Alumina (Al(_2)O(_3))</td>
<td>3970</td>
<td>765</td>
<td>40</td>
</tr>
</tbody>
</table>

### B. Numerical Method

The meshes are produced by GAMBIT software. Computational grid information is imported from GAMBIT into FLUENT and computational mesh is generated using quadrilateral elements. Numerical simulations are performed using FLUENT, a commercially available general-purpose Computational Fluid Dynamics code. This package solves the governing equations using control-volume based technique. The second order upwind scheme was chosen for interpolation of the flow variables. The SIMPLE algorithm [31] has been adapted for the pressure velocity coupling. This problem considers a 2D section of a circular cylinder. For the simulations presented here fine meshes of 62045 cells was used. A sample of the mesh is shown in Fig. 2.

In this domain, quadrilateral cells are used in the regions surrounding the cylinder walls and the rest of the domain. In all simulation, a convergence criterion of \( 1 \times 10^{-6} \) was used for all variables.

### III. RESULTS AND DISCUSSION

Table II presents the details of the grid independence test carried out for \( \text{Re}=200 \). Six types of grids had been used. The difference in grids is based on the interval size of nodes on cylinder surface. Base on variation of lift and drag coefficient and Strouhal number, Grid 5 had been chosen.

#### TABLE II

<table>
<thead>
<tr>
<th>Grid independence test for circular tube at ( \text{Re}=200 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intervalsize of Node on Tube Surface (m)</td>
</tr>
<tr>
<td>Grid 1</td>
</tr>
<tr>
<td>Grid 2</td>
</tr>
<tr>
<td>Grid 3</td>
</tr>
<tr>
<td>Grid 4</td>
</tr>
<tr>
<td>Grid 5</td>
</tr>
<tr>
<td>Grid 6</td>
</tr>
</tbody>
</table>

For the purpose of the validation of the solution procedure, it is essential that numerical simulations be compared with experimental data. Table III compares results of flow parameter of present work with other work in literature. As it is clear from there is a good agreement between Strouhal number, drag and lift coefficient and of present study with others.

#### TABLE III

<table>
<thead>
<tr>
<th>Author Type of Data</th>
<th>( \text{Re}=200 )</th>
<th>( \text{Re}=100 )</th>
<th>( \text{Re}=50 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Present work</td>
<td>Numerical</td>
<td>1.38±0.04</td>
<td>±0.694</td>
</tr>
<tr>
<td>Meneghini et al. [32]</td>
<td>Numerical</td>
<td>1.30±0.05</td>
<td>±0.659</td>
</tr>
<tr>
<td>Ding et al. [33]</td>
<td>Numerical</td>
<td>1.34±0.05</td>
<td>±0.75</td>
</tr>
<tr>
<td>Mahir and Alatc [34]</td>
<td>Numerical</td>
<td>1.176</td>
<td>----</td>
</tr>
<tr>
<td>Williamson [35]</td>
<td>Experimental</td>
<td>----</td>
<td>0.196</td>
</tr>
</tbody>
</table>

Table IV compares the Nusselt number of circular cylinder with the results of [36] and Churchill-Bernstein correlation [37].

#### TABLE IV

<table>
<thead>
<tr>
<th>Author Type of Data Reynolds Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>Present work</td>
</tr>
<tr>
<td>Zukauskas [36]</td>
</tr>
<tr>
<td>Churchill and Bernstein [37]</td>
</tr>
</tbody>
</table>

There is a difference of about 7-12% between the present results and the results of [36] and 3-6% with Churchill-Bernstein equation [37]. It can therefore be concluded that the CFD code can be used to solve the flow field for similar geometries and conditions.

#### A. Effect of Nanoparticle on Fluid Flow

Fig. 3 represents effect of adding nanoparticle on drag coefficient of cylinder. Results show that by increasing volume fraction of Al\(_2\)O\(_3\) drag coefficient increased about 1-4%. Moreover, for a fixed value of volume fraction increasing Reynolds number from 50 to 200 results in 8-11% decrease in the value of drag coefficient.

Oscillation of lift coefficient for \( \text{Re}=100 \) and 200 is presented in Fig. 4. As it is clear, for \( \text{Re}=200 \) by increasing volume fraction from 5% to 20%, the amplitude of oscillation...
of lift coefficient decreases from 0.708 to 0.646. For Re=100 this amplitude decreases from 0.345 to 302. When volume fraction increases the value of density is also increases which results in decrement in the value of lift oscillation.

**Fig. 3** Time histories of drag coefficient of nanofluid passing a circular tube; (a) Re=100 (b) Re=200

**B. Effect of Nanoparticle on Heat Transfer**

Fig. 5 shows unsteady heat transfer of nanofluid for volume fraction of 5% to 20% and Reynolds number 50 to 200. It can be seen that for both value of Reynolds number, by increasing $\phi$ from 5% to 20%, the value of Nusselt number increases.

Effects of adding Al$_2$O$_3$ nanoparticle on heat transfer from a circular cylinder are shown in Fig. 6 for Reynolds number in range of 50 to 200. Results showed for Re=200 increasing volume fraction from 5% to 20% leads to 24% increase in the value of Nusselt number. Moreover, by adding Al$_2$O$_3$ nanoparticle to pure water with 5% of volume fraction, heat transfer from circular cylinder increased up to 10%.

For Re=100 augmentation of volume fraction from 5% to 20% results in 34% increase in the value of Nusselt number. Furthermore, by adding Al2O3 nanoparticle to pure water with only 5% of volume fraction, heat transfer from circular cylinder increased up to 23%. Moreover, for Re=50, Nusselt number increases about 37% by increasing $\phi$ from 5% to 20%. Addition of Al2O3 with 5% of volume fraction increases heat transfer by 26%.

Effect of volume fraction on thermal-hydraulic performance of circular tube for Reynolds number in range of 50 to 200 is presented in Fig. 7. It is clear that by increasing volume fraction from 5% to 20% of nanoparticle, thermal-hydraulic performance of circular tube is increased. This increment is 23%, 24%, and 27% for Reynolds number 50, 100, and 200, respectively.

**Fig. 4** Time histories of lift coefficient of nanofluid passing a circular tube; (a) Re=100 (b) Re=200
Pure water increases the value of drag coefficient about 1-4% and by increasing from 5% to 20%, the amplitude of oscillation of lift coefficient decreases about 8-12%. Furthermore, using nanofluid enhanced heat transfer from circular tube in cross-flow up to 37% and increases thermal-hydraulic performance of circular tube 9% to 36%. These advantages can be used to the increase thermal performance of heat exchanger.

**IV. CONCLUSION**

Unsteady flow and heat transfer from circular tube with constant wall temperature is investigated by using finite volume method. Results show that addition of nanoparticle to pure water increases the value of drag coefficient about 1-4% and by increasing from 5% to 20%, the amplitude of oscillation of lift coefficient decreases about 8-12%.

**REFERENCES**


