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Controller Design for Active Suspension System of ¼ Car with Unknown Mass and Time-Delay

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***Abstract*—**The purpose of this paper is to present a modeling and control of a quarter-car active suspension system with unknown mass, unknown time-delay and road disturbance. The objective of designing the controller is to derive a control law to achieve stability of the system and convergence that can considerably improve ride comfort and road disturbance handling. This is accomplished by using Routh-Hurwitz criterion based on defined parameters. Mathematical proof is given to show the ability of the designed controller to ensure the target of design, implementation with the active suspension system and enhancement dispersion oscillation of the system despite these problems. Simulations were also performed to control quarter car suspension, where the results obtained from these simulations verify the validity of the proposed design.

Keywords—Active suspension system, disturbance rejection, dynamic uncertainty, time-delay.

I. Introduction

Vehicle suspension systems serve a dual purpose. They contribute to road handling and braking devices, increasing safety and driving pleasure. They keep vehicle occupants comfortable and reasonably well isolated from road noise, bumps, and vibrations.

It is the acceleration of the vehicle body that determines ride comfort, the task of the suspension system is to isolate disturbances from the vehicle body, which are caused by uneven road profile. The wheels ability to transfer the contact patch load onto the road could affect the safety of the vehicle while travelling, cornering and maneuvering. The necessity for the vehicle suspension system is to keep the wheels as close as possible to the road surface. The body of the vehicle is mainly isolated from high frequency disturbance of the road by the suspension system.

Passive suspension systems built of springs and dampers have serious limitations. Their parameters have to be selected to achieve a certain level of compromise between road holding, load carrying, and comfort under a wide variety of road conditions. This has motivated extensive research into active and semi-active suspension systems. Numerous researchers have proposed various control strategies to improve the trade-off between ride comfort and road handling that occur when passive car suspension is used. These include LQR method [1], H infinity control [2], sliding mode control (SMC) [3].

In experimentation, the active suspension system also consists of an additional element, which is a sensor. In general, the function of a sensor in active suspension systems is to measure suspension variables such as body velocity, suspension displacement, and wheel velocity. Wheel or body acceleration is also used for measuring pressure and flow rate of the actuator by introducing the appropriate controller into the active suspension system, so that the performance of the system can be improved further [4].

Time-delay inevitably exists in most practical systems. Generally, it is derived from on-line data acquisition, processing, control force calculation and transmission. Neglecting time-delay may cause degradation of control performance or even induce instability of the dynamic system. In recent years, the analysis and design of time-delay systems have received considerable attention from the research community, and some research results have been obtained in application as well as theory fields. For instance, a new successive approximation approach has been proposed by [5] to solve the optimal control problem for discrete-time linear delay systems and linear large-scale systems with small time-delay, respectively. Hence, time delays are unavoidable. In electronic controllers, transfer delays of sensor-controller and controller-actuator are encountered. Recently, issues on network-induced time-delay and sampled-data control problems have attracted widespread attention [6].

In this study, design of stable controllers that achieve stability with minimized overshoot has been implemented within the active suspension system that can considerably improve the ride comfort and road handling, despite the presence of unknown mass, unknown time-delay and the external road disturbance by using Routh-Herwitz criterion.

II. Dynamic Model of Active Suspension System

Fig. 1 demonstrates the quarter car active suspension system. The following are the equations of motion for the active suspension system of the quarter model for a car with time-delay:

$$m_b\ddot{x}_b + c_b(\dot{x}_b - \dot{x}_w) + k_b(x_b - x_w) - u(t-T)f_a = 0 \tag{1}$$

$$\begin{aligned} m_w\ddot{x}_w + c_b(\dot{x}_w - \dot{x}_b) + k_b(x_w - x_b) \\ + k_w(x_w - w) + u(t-T)f_a = 0 \end{aligned} \tag{2}$$

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where m_b and m_w are the masses of the body and wheel. The displacements of wheel and car body are x_w, x_b respectively. The spring coefficients are k_b and k_w. The damper coefficient is c_b and the road disturbance is $w(t)$. A u is a unit-step function, T is unknown time-delay and $f_a(t)$, control force, represents the suspension system control input. Thus, the main objective of this study is to propose a controller, that is robust enough to overcome the mismatched condition and, obviously, the disturbance would not have a significant effect on the system performance for different unknown time-delays. Using a bounded controller $f_a(t)$, implemented within the active suspension, system should considerably improve the ride comfort and road handling, despite the presence of unknown time-delays and external road disturbances.

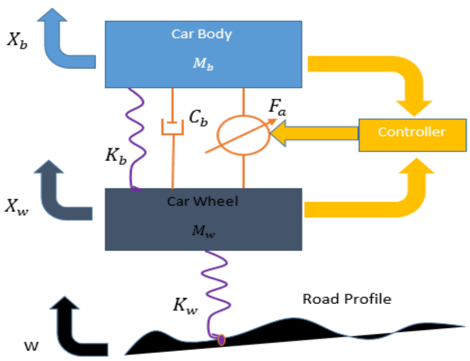

Fig. 1 Active suspension system for a quarter-car

The following assumptions are used throughout this work:

Assumption 1. The parameters m_w, k_b, k_w & c_b are known constants.

Assumption 2. The parameter m_b is an unknown constant assumed bounded by known constants $m_{b1} < m_b < m_{b2}$, where m_{b1} and m_{b2} are known constants.

Assumption 3. The time-delay T is unknown, but bounded by known constants T_1 & T_2 such that $0 < T_1 < T < T_2$.

Assumption 4. The road disturbance w (t) demonstrates a single bump as:

$$w(t) = \begin{cases} \sqrt{\dfrac{a(1-\cos 2t)}{2}} & \tau_1 \le t \le \tau_2 \\ 0 & \text{otherwise} \end{cases} \tag{3}$$

where a is the height of the bump, τ_1 and τ_2 are the lower and the upper time limit of the bump.

Assumption 5. The term e^{-Ts} can be represented by the following Tayler series [7]:

$$e^{-Ts} = 1 - Ts + \frac{T^2 s^2}{2!} - \frac{T^3 s^3}{3!} + - \cdots \tag{4}$$

The higher order terms can be neglected, and (4) is reduced to first order:

$$e^{-Ts} = 1 - Ts \tag{5}$$

III. Main Theorem

The main result of this work will be presented in the following theorem:

Theorem: Under assumptions, 1, 2, 3, and 4, (1) and (2) are stable if we satisfy the control law:

$$f_{a1}(t) = \frac{1}{m_w s^2 + k_w}\{A + B\exp(-\lambda_1 t) + C\exp(-\lambda_2 t) + D\exp(-\lambda_3 t) + E\exp(\lambda_4 t)\} \tag{6}$$

See Appendix for A, B, C, D & E.

By taking the Laplace transformer for (1) & (2)

$$m_b s^2 x_b + c_b s x_b - c_b x_w s + k_b x_b - k_b x_w - F_a e^{-Ts} = 0$$

$$(m_b s^2 + c_b s + k_b) x_b - (c_b s + k_b) x_w - F_a e^{-Ts} = 0 \tag{7}$$

$$\Rightarrow m_w s^2 x_w + c_b s x_w - c_b x_b s + k_b x_w - k_b x_b + k_w x_w - k_w w + F_a e^{-Ts} = 0$$

$$\Rightarrow (-c_b s - k_b) x_b + (m_w s^2 + c_b s + k_b + k_w) x_w - k_w w + F_a e^{-Ts} = 0$$

Then,

$$x_w = \frac{(c_b s + k_b) x_b + k_w w - F_a e^{-Ts}}{m_w s^2 + c_b s + k_b + k_w} \tag{8}$$

Substitute (8) into (7) yields:

$$(m_b s^2 + c_b s + k_b) x_b - (c_b s + k_b) \left[\frac{(c_b s + k_b) x_b + k_w w - F_a e^{-Ts}}{m_w s^2 + c_b s + k_b + k_w}\right] - F_a e^{-Ts} = 0$$

$$\Rightarrow (m_w s^2 + c_b s + k_b + k_w)(m_b s^2 + c_b s + k_b) x_b - (c_b s + k_b)\{(c_b s + k_b) x_b + k_w w - F_a e^{-Ts}\} - (m_w s^2 + c_b s + k_b + k_w) F_a e^{-Ts} = 0$$

$$\Rightarrow (m_w m_b s^4 + c_b m_b s^3 + m_b k_b s^2 + k_w m_b s^2 + m_w c_b s^3 + c_b^2 s^2 + k_b c_b s + k_w c_b s + k_b m_w s^2 + c_b k_b s + k_b^2 + k_b k_w) x_b - (c_b^2 s^2 + c_b k_b s) x_b - (c_b k_w s + k_b k_w) w - (k_b^2 + c_b k_b s) x_b + (c_b s + k_b^2) F_a e^{-Ts} - (m_w s^2 + c_b s + k_b + k_w) F_a e^{-Ts} = 0$$

$$\Rightarrow \{m_w m_b s^4 + (c_b m_b + c_b m_w)s^3 + (k_b m_b + k_w m_b + k_b m_w)s^2 + (c_b k_w)s + k_b k_w\}x_b - (m_w s^2 + k_w)F_a e^{-Ts} - (c_b k_w s + k_b k_w)w = 0 \quad (9)$$

Based on Assumption 4, it necessary employs the approximation $e^{-Ts} \approx 1 - Ts$ in (9) yields:

$$\{m_w m_b s^4 + (c_b m_b + c_b m_w)s^3 + (k_b m_b + k_w m_b + k_b m_w)s^2 + (c_b k_w)s + k_b k_w\}x_b - (m_w s^2 + k_w)(1-Ts)F_a - (c_b k_w s + k_b k_w)w = 0 \quad (10)$$

Then, (10) could be rewritten in the form:

$$c_1 x_b - c_2 F_a - c_3 w = 0 \quad (11)$$

where,

$$c_1 = \{m_w m_b s^4 + (c_b m_b + c_b m_w)s^3 + (k_b m_b + k_w m_b + k_b m_w)s^2 + (c_b k_w)s + k_b k_w\}$$

$$c_2 = (m_w s^2 + k_w) \text{ and } c_3 = (c_b k_w s + k_b k_w)$$

$$E(s) = X_b(s) - R(s) \quad (12)$$

Let $F_a = EF_{a1}$ and substitute it in (11)

$$c_1 x_b - c_2 EF_{a1} - c_3 w = 0 \quad (13)$$

From (13) we can find x_b

$$x_b = \frac{c_2 EF_{a1} + c_3 w}{c_1} \quad (14)$$

By taking the Laplace transform of the controller (6), we will find:

$$c_2 F_{a1} = \frac{c_1(1 - Ts)}{s^5 + 100s^4 + 100s^3 + 100s^2 + 100s} \quad (15)$$

From (12), (14) is represented in the block diagram as shown in Fig. 2.

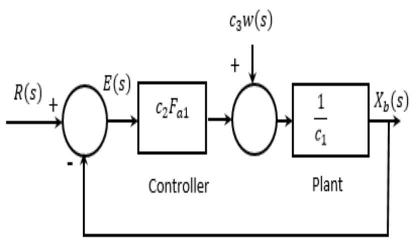

Fig. 2 Block Diagram of (14)

To simplify this analysis, we can reduce a block diagram in Fig. 2 system (14) to a general block diagram as shown in Fig. 3.

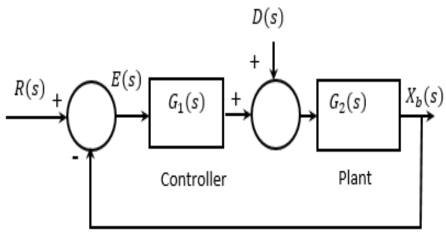

Fig. 3 Block Diagram of (14)

From Fig. 2, the corresponding characteristic equation is:

$$G = G_1 G_2$$

where,

$$1 + GH = 1 + \frac{c_1(1 - Ts)}{s^5 + 100s^4 + 100s^3 + 100s^2 + 100s}[\frac{1}{c_1}]$$

$$1 + GH = 1 + \frac{(1 - Ts)}{s^5 + 100s^4 + 100s^3 + 100s^2 + 100s} \quad (16)$$

Equating (15) to zero, we will find

$$s^5 + 100s^4 + 100s^3 + 100s^2 + 100s + (1 - Ts) = 0$$

$$s^5 + 100s^4 + 100s^3 + 100s^2 + (100 - T)s + 1 = 0 \quad (17)$$

The corresponding Routh table of (17) is constructed as:

s^5	1	100	$100 - T$	1
s^4	100	100	1	0
s^3	99	$99 - T$	1	0
s^2	$\frac{100T}{99}$	$\frac{-T-1}{99-T}$	0	0
s^1	f	1	0	0
s^0	g	0	0	0

Note: See f & g in Appendix.

Since, there is no change in the sign of the first column; the system is stable according to Routh-Hurwitz criterion.

IV. Simulation

In this section, we demonstrate the validity of the proposed controller designed in the Theorem throughout by considering a case study of the active suspension system for the quarter of a car. Based on the Theorem, the dynamics of this case study could be represented by:

$$\{m_w m_b s^4 + (c_b m_b + c_b m_w)s^3 + (k_b m_b + k_w m_b + k_b m_w)s^2 + (c_b k_w)s + k_b k_w\}x_b - (m_w s^2 + k_w)F_a e^{-Ts} - (c_b k_w s + k_b k_w)w = 0 \quad (18)$$

where, m_b and m_w are the masses of the body and wheel, x_b is the displacement of car body, k_b and k_w are the spring coefficients, c_b is the damper coefficient, $w(t)$ is the road disturbance, $F_a(t)$, control force, is suggested to be the suspension system control input. For the purpose of simulation, it necessary to consider the values, a=10cm $\tau_1 = 0 \,\&\, \tau_2 = 10$, and the dynamics parameters demonstrated in Table I [8].

TABLE I
PARAMETERS VALUE FOR THE QUARTER CAR MODEL

Mass for car body, m_{b1} & m_{b2}	(300 & 500) kg
Mass for car wheel, m_w	50 kg
Stiffness of car body spring, k_b	16812 N/m
Stiffness of car wheel spring, k_w	190000 N/m
Damping of the damper, c_b	1000 Ns/m

The aim of this study is to employ the design of the control law to implement within the active car suspension system that could considerably improve the ride comfort and road handling. Figs. 4, 6 and 7 show the time-response of (14) by using control law (6). It can be clearly seen that the control law (6) achieves a good tracking performance and successfully drives the disturbances overshoot to the desired zero value with a fast rate of convergence using a bounded controller f_{a1}, which is applied between the wheel and car body. In addition, the passive suspension for quarter car travel produces an overshoot greater than 30 cm, which is illustrated in Fig. 5. Fig. 6, provides the time-response of (14) for different values of m_b (300 kg, 400 kg, 500 kg) with different values of time-delays T (1s, 1.5s, 2s). It is strongly apparent from this figure that there is a difference between the overshoot. This is might be on account, of the effect of change in masses, when mass increased the overshoot will increase so. Fig. 7 presents the comparison between the passive and active time-response of (14) for three different values of time-delay and three different values of masses by using the proposed controller design. As shown in this figure, the overshoot of the travel active suspension significantly drops; it will become lower than (3cm). Also within a few seconds, faster suppression of the oscillation is achieved to the desired value (zero) with a good rate of convergence, although this presents critical issues, in term of the range values of T and different values of m_b. Fig. 8 highlights the interesting decreases in the car body acceleration of active suspension in comparison with passive through different values of time-delay and different values of masses by using the suggested controller. Subsequently, it is necessary to know the behavior of the car wheel travel due to limited allowance space. Therefore, Fig. 9 provides the comparison between the deflection of the wheel car for active and passive suspension in cases with changing values of time-delay and masses. Finally, the wheel vibration is damped and any dangerous lifting of the wheels is avoided, despite the presence of time-delay, different masses and road disturbance.

V. CONCLUSIONS

In this study, a controller is designed by using the Routh-Hurwitz stability criterion, which has been executed for the active car suspension system. The results of the presented mathematical model and simulation show a significant improvement in the performance and disturbance absorption for a system with unknown time-delay, different masses and road disturbance. The control law (6) designed by using Routh-Hurwitz criterion has been implemented successfully to the active car suspension system, which is robust in compensating for the disturbance in the system and can significantly improve ride comfort and road handling. For these reasons, the Routh-Hurwitz criterion control is recommended in solving a time delay system with uncertainties.

For future work, a study will be considered into the effect of contact patch load between tyre and road. This could influence the stability and ride comfort of vehicles. This study will cover the passive, semi-active and active suspension with different conditions, such as tyre characteristic or tyre pressure with the scope of steps input.

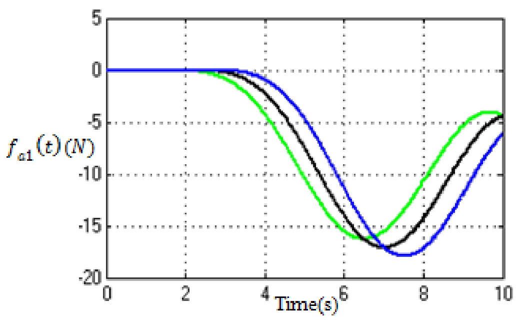

Fig. 4 Control force supposed to be the system with three different values for time-delay (1s, 1.5s, 2s)

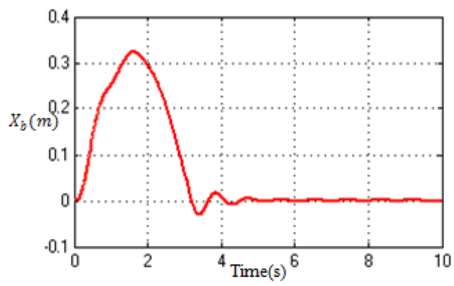

Fig. 5 Passive suspension travel

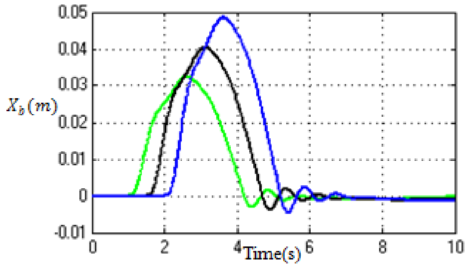

Fig. 6 Active suspension time travel with three different values of time-delay and three different values of the mass

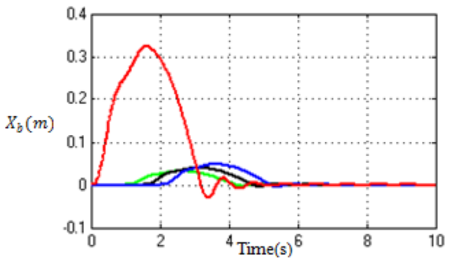

Fig. 7 Suspension time travel between active and Passive with three different values for time-delay and three different values for the mass

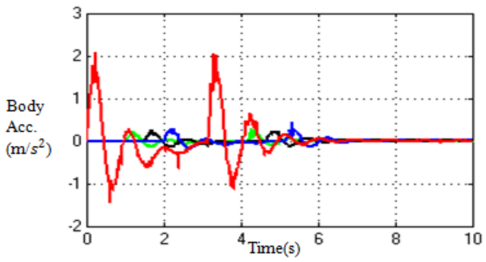

Fig. 8 Body acceleration between active and Passive with three different values for time-delay and three different values for the mass

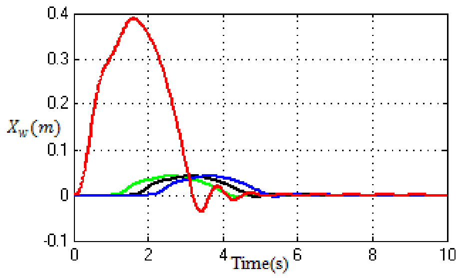

Fig. 9 Wheel deflection between active and passive with three different values for time-delay and three different values for the mass

Appendix

The parameters of control law (6) are as:

$$A = \frac{k_b k_w}{\lambda_1 \lambda_2 \lambda_3 \lambda_4}$$

$$B = \{\frac{-m_w m_b \lambda_1^4}{-\lambda_1(-\lambda_1+\lambda_2)(-\lambda_1+\lambda_3)(-\lambda_1+\lambda_4)} + \frac{-(c_b m_b + c_b m_w)\lambda_1^3}{-\lambda_2(-\lambda_2+\lambda_1)(-\lambda_2+\lambda_3)(-\lambda_2+\lambda_4)} + \frac{(k_b m_b + k_w m_b + k_b m_w)\lambda_1^2}{-\lambda_1(-\lambda_1+\lambda_2)(-\lambda_1+\lambda_3)(-\lambda_1+\lambda_4)} + \frac{-c_b k_w \lambda_1 + k_b k_w}{-\lambda_1(-\lambda_1+\lambda_2)(-\lambda_1+\lambda_3)(-\lambda_1+\lambda_4)}\}(1-T\lambda_1)$$

$$C = \{\frac{-m_w m_b \lambda_2^4}{-\lambda_1(-\lambda_1+\lambda_2)(-\lambda_1+\lambda_3)(-\lambda_1+\lambda_4)} + \frac{-(c_b m_b + c_b m_w)\lambda_2^3}{-\lambda_2(-\lambda_2+\lambda_1)(-\lambda_2+\lambda_3)(-\lambda_2+\lambda_4)} + \frac{(k_b m_b + k_w m_b + k_b m_w)\lambda_2^2}{-\lambda_1(-\lambda_1+\lambda_2)(-\lambda_1+\lambda_3)(-\lambda_1+\lambda_4)} + \frac{-c_b k_w \lambda_2 + k_b k_w}{-\lambda_1(-\lambda_1+\lambda_2)(-\lambda_1+\lambda_3)(-\lambda_1+\lambda_4)}\}(1-T\lambda_2)$$

$$D = \{\frac{-m_w m_b \lambda_3^4}{-\lambda_1(-\lambda_1+\lambda_2)(-\lambda_1+\lambda_3)(-\lambda_1+\lambda_4)} + \frac{-(c_b m_b + c_b m_w)\lambda_3^3}{-\lambda_2(-\lambda_2+\lambda_1)(-\lambda_2+\lambda_3)(-\lambda_2+\lambda_4)} + \frac{(k_b m_b + k_w m_b + k_b m_w)\lambda_3^2}{-\lambda_1(-\lambda_1+\lambda_2)(-\lambda_1+\lambda_3)(-\lambda_1+\lambda_4)} + \frac{-c_b k_w \lambda_3 + k_b k_w}{-\lambda_1(-\lambda_1+\lambda_2)(-\lambda_1+\lambda_3)(-\lambda_1+\lambda_4)}\}(1-T\lambda_3)$$

$$E = \{\frac{-m_w m_b \lambda_4^4}{-\lambda_1(-\lambda_1+\lambda_2)(-\lambda_1+\lambda_3)(-\lambda_1+\lambda_4)} + \frac{-(c_b m_b + c_b m_w)\lambda_4^3}{-\lambda_2(-\lambda_2+\lambda_1)(-\lambda_2+\lambda_3)(-\lambda_2+\lambda_4)} + \frac{(k_b m_b + k_w m_b + k_b m_w)\lambda_4^2}{-\lambda_1(-\lambda_1+\lambda_2)(-\lambda_1+\lambda_3)(-\lambda_1+\lambda_4)} + \frac{-c_b k_w \lambda_4 + k_b k_w}{-\lambda_1(-\lambda_1+\lambda_2)(-\lambda_1+\lambda_3)(-\lambda_1+\lambda_4)}\}(1-T\lambda_4)$$

In addition, the parameters for Routh table are:

$$f = \frac{(10000T^4 - 990000T^3 + 98000100T^2 + 980100T)}{(99-T)}$$

$$g = \frac{(f * \frac{(-T-1)}{(99-T)}) - \frac{100T}{99}}{f}$$

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