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Restrictedly-Regular Map Representation of n-Dimensional Abstract Polytopes

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Abstract : Regularity has often been present in the form of regular polyhedra or tessellations; classical examples are the nine regular polyhedra consisting of the five Platonic solids (regular convex polyhedra) and the four Kleper-Poinsot polyhedra. These polytopes can be seen as regular maps. Maps are cellular embeddings of graphs (with possibly multiple edges, loops or dangling edges) on compact connected (closed) surfaces with or without boundary. The n-dimensional abstract polytopes, particularly the regular ones, have gained popularity over recent years. The main focus of research has been their symmetries and regularity. Planification of polyhedra helps its spatial construction, yet it destroys its symmetries. To our knowledge there is no “planification” for n-dimensional polytopes. However we show that it is possible to make a “surfacification” of the n-dimensional polytope, that is, it is possible to construct a restrictedly-marked map representation of the abstract polytope on some surface that describes its combinatorial structures as well as all of its symmetries. We also show that there are infinitely many ways to do this; yet there is one that is more natural that describes reflections on the sides ((n−1)-faces) of n-simplices with reflections on the sides of n-polygons. We illustrate this construction with the 4-tetrahedron (a regular 4-polytope with automorphism group of size 120) and the 4-cube (a regular 4-polytope with automorphism group of size 384).

Keywords : abstract polytope, automorphism group, N-simplicies, symmetry
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