Conditions on Expressing a Matrix as a Sum of $\alpha$-Involutions

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Abstract: Let $F$ be $C$ or $R$, where $C$ and $R$ are the set of complex numbers and real numbers, respectively, and $n$ be a natural number. An $n$-by-$n$ matrix $A$ over the field $F$ is called an $\alpha$-involutory matrix or an $\alpha$-involution if there exists an $\alpha$ in the field such that the square of the matrix is equal to $\alpha I$, where $I$ is the $n$-by-$n$ identity matrix. If $\alpha$ is a complex number or a nonnegative real number, then an $n$-by-$n$ matrix $A$ over the field $F$ can be written as a sum of $n$-by-$n$ $\alpha$-involutory matrices over the field $F$ if and only if the trace of that matrix is an integral multiple of the square root of $\alpha$. Meanwhile, if $\alpha$ is a negative real number, then a $2n$-by-$2n$ matrix $A$ over $R$ can be written as a sum of $2n$-by-$2n$ $\alpha$-involutory matrices over $R$ if and only if the trace of the matrix is zero. Some other properties of $\alpha$-involutory matrices are also determined.

Keywords: $\alpha$-involutory Matrices, sum of $\alpha$-involutory Matrices, Trace, Matrix Theory

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