World Academy of Science, Engineering and Technology
International Journal of Mechanical and Industrial Engineering
Vol:12, No:09, 2018

Q-Efficient Solutions of Vector Optimization via Algebraic Concepts

Authors : Elham Kiyani

Abstract : In this paper, we first introduce the concept of Q-efficient solutions in a real linear space not necessarily endowed with a topology, where Q is some nonempty (not necessarily convex) set. We also used the scalarization technique including the Gerstewitz function generated by a nonconvex set to characterize these Q-efficient solutions. The algebraic concepts of interior and closure are useful to study optimization problems without topology. Studying nonconvex vector optimization is valuable since topological interior is equal to algebraic interior for a convex cone. So, we use the algebraic concepts of interior and closure to define Q-weak efficient solutions and Q-Henig proper efficient solutions of set-valued optimization problems, where Q is not a convex cone. Optimization problems with set-valued maps have a wide range of applications, so it is expected that there will be a useful analytical tool in optimization theory for set-valued maps. These kind of optimization problems are closely related to stochastic programming, control theory, and economic theory. The paper focus on nonconvex problems, the results are obtained by assuming generalized non-convexity assumptions on the data of the problem. In convex problems, main mathematical tools are convex separation theorems, alternative theorems, and algebraic counterparts of some usual topological concepts, while in nonconvex problems, we need a nonconvex separation function. Thus, we consider the Gerstewitz function generated by a general set in a real linear space and re-examine its properties in the more general setting. A useful approach for solving a vector problem is to reduce it to a scalar problem. In general, scalarization means the replacement of a vector optimization problem by a suitable scalar problem which tends to be an optimization problem with a real valued objective function. The Gerstewitz function is well known and widely used in optimization as the basis of the scalarization. The essential properties of the Gerstewitz function, which are well known in the topological framework, are studied by using algebraic counterparts rather than the topological concepts of interior and closure. Therefore, properties of the Gerstewitz function, when it takes values just in a real linear space are studied, and we use it to characterize Q-efficient solutions of vector problems whose image space is not endowed with any particular topology. Therefore, we deal with a constrained vector optimization problem in a real linear space without assuming any topology, and also Q-weak efficient and Q-proper efficient solutions in the senses of Henig are defined. Moreover, by means of the Gerstewitz function, we provide some necessary and sufficient optimality conditions for set-valued vector optimization problems.

Keywords : algebraic interior, Gerstewitz function, vector closure, vector optimization
Conference Title : ICMAO 2018 : International Conference on Mechanical Analysis and Optimization
Conference Location : Vancouver, Canada
Conference Dates : September 17-18, 2018