Nonlinear Evolution on Graphs

Authors : Benniche Omar

Abstract : We are concerned with abstract fully nonlinear differential equations having the form y'(t)=Ay(t)+f(t,y(t)) where A is an m—dissipative operator (possibly multi—valued) defined on a subset D(A) of a Banach space X with values in X and f is a given function defined on I×X with values in X. We consider a graph K in I×X. We recall that K is said to be viable with respect to the above abstract differential equation if for each initial data in K there exists at least one trajectory starting from that initial data and remaining in K at least for a short time. The viability problem has been studied by many authors by using various techniques and frames. If K is closed, it is shown that a tangency condition, which is mainly linked to the dynamic, is crucial for viability. In the case when X is infinite dimensional, compactness and convexity assumptions are needed. In this paper, we are concerned with the notion of near viability for a given graph K with respect to y'(t)=Ay(t)+f(t,y(t)). Roughly speaking, the graph K is said to be near viable with respect to y'(t)=Ay(t)+f(t,y(t)), if for each initial data in K there exists at least one trajectory remaining arbitrary close to K at least for short time. It is interesting to note that the near viability is equivalent to an appropriate tangency condition under mild assumptions on the dynamic. Adding natural convexity and compactness assumptions on the dynamic, we may recover the (exact) viability. Here we investigate near viability for a graph K in I×X with respect to y'(t)=Ay(t)+f(t,y(t)) where A and f are as above. We emphasis that the t—dependence on the perturbation f leads us to introduce a new tangency concept. In the base of a tangency conditions expressed in terms of that tangency concept, we formulate criteria for K to be near viable with respect to y'(t)=Ay(t)+f(t,y(t)). As application, an abstract null—controllability theorem is given.

Keywords : abstract differential equation, graph, tangency condition, viability
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