

## High Accuracy Analytic Approximation for Special Functions Applied to Bessel Functions $J_0(x)$ and Its Zeros

**Authors :** Fernando Maass, Pablo Martin, Jorge Olivares

**Abstract :** The Bessel function  $J_0(x)$  is very important in Electrodynamics and Physics, as well as its zeros. In this work, a method to obtain high accuracy approximation is presented through an application to that function. In most of the applications of this function, the values of the zeros are very important. In this work, analytic approximations for this function have been obtained valid for all positive values of the variable  $x$ , which have high accuracy for the function as well as for the zeros. The approximation is determined by the simultaneous use of the power series and asymptotic expansion. The structure of the approximation is a combination of two rational functions with elementary functions as trigonometric and fractional powers. Here we use the Pade method, rational functions are used, but now they are combined with elementary functions such as fractional powers, hyperbolic or trigonometric functions, and others. The reason for this is that now power series of the exact function are used, but together with the asymptotic expansion, which usually includes fractional powers, trigonometric functions and other type of elementary functions. The approximation must be a bridge between both expansions, and this can not be accomplished using only with rational functions. In the simplest approximation using 4 parameters the maximum absolute error is less than 0.006 at  $x \sim 4.9$ . In this case also the maximum relative error for the zeros is less than 0.003 which is for the second zero, but that value decreases rapidly for the other zeros. The same kind of behaviour happens for the relative error of the maximum and minimum of the functions. Approximations with higher accuracy and more parameters will be also shown. All the approximations are valid for any positive value of  $x$ , and they can be calculated easily.

**Keywords :** analytic approximations, asymptotic approximations, Bessel functions, quasirational approximations

**Conference Title :** ICPM 2018 : International Conference on Physics and Mathematics

**Conference Location :** Boston, United States

**Conference Dates :** April 23-24, 2018