Kirchoff Type Equation Involving the p-Laplacian on the Sierpinski Gasket Using Nehari Manifold Technique

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Abstract : In this paper, we will discuss the existence of weak solutions of the Kirchhoff type boundary value problem on the Sierpinski gasket. Where S denotes the Sierpinski gasket in R^2 and S_0 is the intrinsic boundary of the Sierpinski gasket. M: R → R is a positive function and h: S × R → R is a suitable function which is a part of our main equation. Δp denotes the p-Laplacian, where $p > 1$. First of all, we will define a weak solution for our problem and then we will show the existence of at least two solutions for the above problem under suitable conditions. There is no well-known concept of a generalized derivative of a function on a fractal domain. Recently, the notion of differential operators such as the Laplacian and the p-Laplacian on fractal domains has been defined. We recall the result first then we will address the above problem. In view of literature, Laplacian and p-Laplacian equations are studied extensively on regular domains (open connected domains) in contrast to fractal domains. In fractal domains, people have studied Laplacian equations more than p-Laplacian probably because in that case, the corresponding function space is reflexive and many minimax theorems which work for regular domains is applicable there which is not the case for the p-Laplacian. This motivates us to study equations involving p-Laplacian on the Sierpinski gasket. Problems on fractal domains lead to nonlinear models such as reaction-diffusion equations on fractals, problems on elastic fractal media and fluid flow through fractal regions etc. We have studied the above p-Laplacian equations on the Sierpinski gasket using fibering map technique on the Nehari manifold. Many authors have studied the Laplacian and p-Laplacian equations on regular domains using this Nehari manifold technique. In general Euler functional associated with such a problem is Frechet or Gateaux differentiable. So, a critical point becomes a solution to the problem. Also, the function space they consider is reflexive and hence we can extract a weakly convergent subsequence from a bounded sequence. But in our case neither the Euler functional is differentiable nor the function space is known to be reflexive. Overcoming these issues we are still able to prove the existence of at least two solutions of the given equation.

Keywords : Euler functional, p-Laplacian, p-energy, Sierpinski gasket, weak solution
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