

Building 1-Well-Covered Graphs by Corona, Join, and Rooted Product of Graphs

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Abstract : A graph is well-covered if all its maximal independent sets are of the same size. A well-covered graph is 1-well-covered if deletion of every vertex of the graph leaves it well-covered. It is known that a graph without isolated vertices is 1-well-covered if and only if every two disjoint independent sets are included in two disjoint maximum independent sets. Well-covered graphs are related to combinatorial commutative algebra (e.g., every Cohen-Macaulay graph is well-covered, while each Gorenstein graph without isolated vertices is 1-well-covered). Our intent is to construct several infinite families of 1-well-covered graphs using the following known graph operations: corona, join, and rooted product of graphs. Adopting some known techniques used to advantage for well-covered graphs, one can prove that: if the graph G has no isolated vertices, then the corona of G and H is 1-well-covered if and only if H is a complete graph of order two at least; the join of the graphs G and H is 1-well-covered if and only if G and H have the same independence number and both are 1-well-covered; if H satisfies the property that every three pairwise disjoint independent sets are included in three pairwise disjoint maximum independent sets, then the rooted product of G and H is 1-well-covered, for every graph G . These findings show not only how to generate some more families of 1-well-covered graphs, but also that, to this aim, sometimes, one may use graphs that are not necessarily 1-well-covered.

Keywords : maximum independent set, corona, concatenation, join, well-covered graph

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