World Academy of Science, Engineering and Technology
International Journal of Mathematical and Computational Sciences
Vol:12, No:01, 2018

Asymptotic Analysis of the Viscous Flow through a Pipe and the Derivation of the Darcy-Weisbach Law

Authors : Eduard Marusic-Paloka

Abstract : The Darcy-Weisbach formula is used to compute the pressure drop of the fluid in the pipe, due to the friction against the wall. Because of its simplicity, the Darcy-Weisbach formula became widely accepted by engineers and is used for laminar as well as the turbulent flows through pipes, once the method to compute the mysterious friction coefficient was derived. Particularly in the second half of the 20th century. Formula is empiric, and our goal is to derive it from the basic conservation law, via rigorous asymptotic analysis. We consider the case of the laminar flow but with significant Reynolds number. In case of the perfectly smooth pipe, the situation is trivial, as the Navier-Stokes system can be solved explicitly via the Poiseuille formula leading to the friction coefficient in the form 64/Re. For the rough pipe, the situation is more complicated and some effects of the roughness appear in the friction coefficient. We start from the Navier-Stokes system in the pipe with periodically corrugated wall and derive an asymptotic expansion for the pressure and for the velocity. We use the homogenization techniques and the boundary layer analysis. The approximation derived by formal analysis is then justified by rigorous error estimate in the norm of the appropriate Sobolev space, using the energy formulation and classical a priori estimates for the Navier-Stokes system. Our method leads to the formula for the friction coefficient. The formula involves resolution of the appropriate boundary layer problems, namely the boundary value problems for the Stokes system in an infinite band, that needs to be done numerically. However, theoretical analysis characterising their nature can be done without solving them.

Keywords : Darcy-Weisbach law, pipe flow, rough boundary, Navier law
Conference Title : ICIAM 2018 : International Conference on Industrial and Applied Mathematics
Conference Location : Paris, France
Conference Dates : January 25-26, 2018

Open Science Index, Mathematical and Computational Sciences Vol:12, No:01, 2018 publications.waset.org/abstracts/84028.pdf