An Adjoint-Based Method to Compute Derivatives with Respect to Bed Boundary Positions in Resistivity Measurements

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Abstract : Resistivity measurements are used to characterize the Earth's subsurface. They are categorized into two different groups: (a) those acquired on the Earth's surface, for instance, controlled source electromagnetic (CSEM) and Magnetotellurics (MT), and (b) those recorded with borehole logging instruments such as Logging-While-Drilling (LWD) devices. LWD instruments are mostly used for geo-steering purposes, i.e., to adjust dip and azimuthal angles of a well trajectory to drill along a particular geological target. Modern LWD tools measure all nine components of the magnetic field corresponding to three orthogonal transmitter and receiver orientations. In order to map the Earth's subsurface and perform geo-steering, we invert measurements using a gradient-based method that utilizes the derivatives of the recorded measurements with respect to the inversion variables. For resistivity measurements, these inversion variables are usually the constant resistivity value of each layer and the bed boundary positions. It is well-known how to compute derivatives with respect to the constant resistivity value of each layer using semi-analytic or numerical methods. However, similar formulas for computing the derivatives with respect to bed boundary positions are unavailable. The main contribution of this work is to provide an adjoint-based formulation for computing derivatives with respect to the bed boundary positions. The key idea to obtain the aforementioned adjoint state formulations for the derivatives is to separate the tangential and normal components of the field and treat them differently. This formulation allows us to compute the derivatives faster and more accurately than with traditional finite differences approximations. In the presentation, we shall first derive a formula for computing the derivatives with respect to the bed boundary positions for the potential equation. Then, we shall extend our formulation to 3D Maxwell's equations. Finally, by considering a 1D domain and reducing the dimensionality of the problem, which is a common practice in the inversion of resistivity measurements, we shall derive a formulation to compute the derivatives of the measurements with respect to the bed boundary positions using a 1.5D variational formulation. Then, we shall illustrate the accuracy and convergence properties of our formulations by comparing numerical results with the analytical derivatives for the potential equation. For the 1.5D Maxwell's system, we shall compare our numerical results based on the proposed adjoint-based formulation vs those obtained with a traditional finite difference approach. Numerical results shall show that our proposed adjoint-based technique produces enhanced accuracy solutions while its cost is negligible, as opposed to the finite difference approach that requires the solution of one additional problem per derivative.

Keywords : inverse problem, bed boundary positions, electromagnetism, potential equation