A Dynamic Symplectic Manifold Analysis for Wave Propagation in Porous Media

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Abstract : This study aims to understand with more amplitude and clarity the behavior of a porous medium where a pressure wave travels, translated into relative displacements inside the material, using mathematical tools derived from topology and symplectic geometry. The paper starts with a given partial differential equation based on the continuity and conservation theorems to describe the traveling wave through the porous body. A solution for this equation is proposed after all boundary, and initial conditions are fixed, and it's accepted that the solution lies in a manifold U of purely spatial dimensions and that is embedded in the Real n-dimensional manifold, with spatial and kinetic dimensions. It's shown that the U manifold of lower dimensions than IRna, where it is embedded, inherits properties of the vector spaces existing inside the topology it lies on. Then, a second manifold (U*), embedded in another space called IRnb of stress dimensions, is proposed and there's a non-degenerative function that maps it into the U manifold. This relation is proved as a transformation in between two corresponding admissible solutions of the differential equation in distinct dimensions and properties, leading to a more visual and intuitive understanding of the whole dynamic process of a stress wave through a porous medium and also highlighting the dimensional invariance of Terzaghi's theory for any coordinate system.

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Keywords : poremechanics, soil dynamics, symplectic geometry, wave propagation

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