Analytical and Numerical Investigation of Friction-Restricted Growth and Buckling of Elastic Fibers

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Abstract : The quasi-static growth of elastic fibers is studied in the presence of distributed contact with an immobile surface, subject to isotropic dry or viscous friction. Unlike classical problems of elastic stability modelled by autonomous dynamical systems with multiple time scales (slowly varying bifurcation parameter, and fast system dynamics), this problem can only be formulated as a non-autonomous system without time scale separation. It is found that the fibers initially converge to a trivial, straight configuration, which is later replaced by divergence reminiscent of buckling phenomena. In order to capture the loss of stability, a new definition of exponential stability against infinitesimal perturbations for systems defined over finite time intervals is developed. A semi-analytical method for the determination of the critical length based on eigenvalue analysis is proposed. The post-critical behavior of the fibers is studied numerically by using variational methods. The emerging post-critical shapes and the asymptotic behavior as length goes to infinity are identified for simple spatial distributions of growth. Comparison with physical experiments indicates reasonable accuracy of the theoretical model. Some applications from modeling plant root growth to the design of soft manipulators in robotics are briefly discussed.