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Ferromagnetic Potts Models with Multi Site Interaction

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Abstract : The Potts model has been widely explored in the literature for the last few decades. While many analytical and numerical results concern with the traditional two site interaction model in various geometries and dimensions, little is yet known about models where more than two spins simultaneously interact. We consider a ferromagnetic four site interaction Potts model on the square lattice (FFPS), where the four spins reside in the corners of an elementary square. Each spin can take an integer value 1,2,...,q. We write the partition function as a sum over clusters consisting of monochromatic faces. When the number of faces becomes large, tracing out spin configurations is equivalent to enumerating large lattice animals. It is known that the asymptotic number of animals with k faces is governed by λ^k, with $\lambda \approx 4.0626$. Based on this observation, systems with $q < 4$ and $q > 4$ exhibit a second and first order phase transitions, respectively. The transition nature of the $q = 4$ case is borderline. For any q, a critical giant component (GC) is formed. In the finite order case, GC is simple, while it is fractal when the transition is continuous. Using simple equilibrium arguments, we obtain a (zero order) bound on the transition point. It is claimed that this bound should apply for other lattices as well. Next, taking into account higher order sites contributions, the critical bound becomes tighter. Moreover, for $q > 4$, if corrections due to contributions from small clusters are negligible in the thermodynamic limit, the improved bound should be exact. The improved bound is used to relate the critical point to the finite correlation length. Our analytical predictions are confirmed by an extensive numerical study of FFPS, using the Wang-Landau method. In particular, the q=4 marginal case is supported by a very ambiguous pseudo-critical finite size behavior.

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