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Comparison of Finite Difference Schemes for Numerical Study of Ripa Model

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Abstract : The river and lakes flows are modeled mathematically by shallow water equations that are depth-averaged Reynolds Averaged Navier-Stokes equations under Boussinesq approximation. The temperature stratification dynamics influence the water quality and mixing characteristics. It is mainly due to the atmospheric conditions including air temperature, wind velocity, and radiative forcing. The experimental observations are commonly taken along vertical scales and are not sufficient to estimate small turbulence effects of temperature variations induced characteristics of shallow flows. Wind shear stress over the water surface influence flow patterns, heat fluxes and thermodynamics of water bodies as well. Hence it is crucial to couple temperature gradients with shallow water model to estimate the atmospheric effects on flow patterns. The Ripa system has been introduced to study ocean currents as a variant of shallow water equations with addition of temperature variations within the flow. Ripa model is a hyperbolic system of partial differential equations because all the eigenvalues of the system's Jacobian matrix are real and distinct. The time steps of a numerical scheme are estimated with the eigenvalues of the system. The solution to Riemann problem of the Ripa model is composed of shocks, contact and rarefaction waves. Solving Ripa model with Riemann initial data with the central schemes is difficult due to the eigen structure of the system.This works presents the comparison of four different finite difference schemes for the numerical solution of Riemann problem for Ripa model. These schemes include Lax-Friedrichs, Lax-Wendroff, MacCormack scheme and a higher order finite difference scheme with WENO method. The numerical flux functions in both dimensions are approximated according to these methods. The temporal accuracy is achieved by employing TVD Runge Kutta method. The numerical tests are presented to examine the accuracy and robustness of the applied methods. It is revealed that Lax-Freidrichs scheme produces results with oscillations while Lax-Wendroff and higher order difference scheme produce quite better results.

Keywords : finite difference schemes, Riemann problem, shallow water equations, temperature gradients
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