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High Accuracy Analytic Approximations for Modified Bessel Functions $I_0(x)$

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Abstract : A method to obtain analytic approximations for special function of interest in engineering and physics is described here. Each approximate function will be valid for every positive value of the variable and accuracy will be high and increasing with the number of parameters to determine. The general technique will be shown through an application to the modified Bessel function of order zero, $I_0(x)$. The form and the calculation of the parameters are performed with the simultaneous use of the power series and asymptotic expansion. As in Padé method rational functions are used, but now they are combined with other elementary functions as; fractional powers, hyperbolic, trigonometric and exponential functions, and others. The elementary function is determined, considering that the approximate function should be a bridge between the power series and the asymptotic expansion. In the case of the $I_0(x)$ function two analytic approximations have been already determined. The simplest one is $(1+x^2/4)^{-1/4}(1+0.24273x^2)\cosh(x)/(1+0.43023x^2)$. The parameters of $I_0(x)$ were determined using the leading term of the asymptotic expansion and two coefficients of the power series, and the maximum relative error is 0.05. In a second case, two terms of the asymptotic expansion were used and 4 of the power series and the maximum relative error is 0.001 at $x\approx 9.5$. Approximations with much higher accuracy will be also shown. In conclusion a new technique is described to obtain analytic approximations to some functions of interest in sciences, such that they have a high accuracy, they are valid for every positive value of the variable, they can be integrated and differentiated as the usual, functions, and furthermore they can be calculated easily even with a regular pocket calculator.

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