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A Fundamental Functional Equation for Lie Algebras

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Abstract : Inspired by the so called Jacobi Identity (x y) z + (y z) x + (z x) y = 0, the following class of functional equations EQ I: F [F (x, y), z] + F [F (y, z), x] + F [F (z, x), y] = 0 is proposed, researched and generalized. Research methodologies begin with classical methods for functional equations, then evolve into discovering of any implicit algebraic structures. One of this paper's major findings is that EQ I, under two additional conditions F (x, x) = 0 and F (x, y) + F (y, x) = 0, proves to be a fundamental functional equation for Lie Algebras. Existence of non-trivial solutions for EQ I can be proven by defining F (p, q) = [p q] = pq –qp, where p and q are quaternions, and pq is the quaternion product of p and q. EQ I can be generalized to the following class of functional equations EQ II: F [G (x, y), z] + F [G (y, z), x] + F [G (z, x), y] = 0. Concluding Statement: With a major finding proven, and non-trivial solutions derived, this research paper illustrates and provides a new functional equation scheme for studies in two major areas: (1) What underlying algebraic structures can be defined and/or derived from EQ I or EQ II? (2) What conditions can be imposed so that conditional general solutions to EQ I and EQ II can be found, investigated and applied?

Keywords : fundamental functional equation, generalized functional equations, Lie algebras, quaternions
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