

## An Equivalence between a Harmonic Form and a Closed Co-Closed Differential Form in $L^q$ and Non- $L^q$ Spaces

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**Abstract :** An equivalent relation between a harmonic form and a closed co-closed form is established on a complete non-compact manifold. This equivalence has been generalized for a differential  $k$ -form  $\omega$  from  $L^q$  spaces to non- $L^q$  spaces when  $q=2$  in the context of  $p$ -balanced growth where  $p=2$ . Especially for a simple differential  $k$ -form on a complete non-compact manifold, the equivalent relation has been verified with the extended scope of  $q$  for from finite  $q$ -energy in  $L^q$  spaces to infinite  $q$ -energy in non- $L^q$  spaces when with 2-balanced growth. Generalized Hadamard Theorem, Cauchy-Schwarz Inequality, and Calculus skills including Integration by Parts as well as Convergent Series have been applied as estimation techniques to evaluate growth rates for a differential form. In particular, energy growth rates as indicated by an appropriate power range in a selected test function lead to a balance between a harmonic differential form and a closed co-closed differential form. Research ideas and computational methods in this paper could provide an innovative way in the study of broadening  $L^q$  spaces to non- $L^q$  spaces with a wide variety of infinite energy growth for a differential form.

**Keywords :** closed forms, co-closed forms, harmonic forms,  $L^q$  spaces,  $p$ -balanced growth, simple differential  $k$ -forms

**Conference Title :** ICDGA 2018 : International Conference on Differential Geometry and Applications

**Conference Location :** New York, United States

**Conference Dates :** June 03-04, 2018