Deciding Graph Non-Hamiltonicity via a Closure Algorithm

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Abstract : We present an heuristic algorithm that decides graph non-Hamiltonicity. All graphs are directed, each undirected edge regarded as a pair of counter directed arcs. Each of the n! Hamilton cycles in a complete graph on n+1 vertices is mapped to an n-permutation matrix P where p(u,i)=1 if and only if the ith arc in a cycle enters vertex u, starting and ending at vertex n+1. We first create exclusion set E by noting all arcs (u, v) not in G, sufficient to code precisely all cycles excluded from G i.e. cycles not in G use at least one arc not in G. Members are pairs of components of P, {p(u,i),p(v,i+1)}, i=1, n-1. A doubly stochastic-like relaxed LP formulation of the Hamilton cycle decision problem is constructed. Each {p(u,i),p(v,i+1)} in E is coded as variable q(u,i,v,i+1)=0 i.e. shrinks the feasible region. We then implement the Weak Closure Algorithm (WCA) that tests necessary conditions of a matching, together with Boolean closure to decide 0/1 variable assignments. Each {p(u,i),p(v,j)} not in E is tested for membership in E, and if possible, added to E (q(u,i,v,j)=0) to iteratively maximize |E|. If the WCA constructs E to be maximal, the set of all {p(u,i),p(v,j)}, then G is decided non-Hamiltonian. Only non-Hamiltonian G share this maximal property. Ten non-Hamiltonian graphs (10 through 104 vertices) and 2000 randomized 31 vertex non-Hamiltonian graphs are tested and correctly decided non-Hamiltonian. For Hamiltonian G, the complement of E covers a matching, perhaps useful in searching for cycles. We also present an example where the WCA fails.

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