

Regularity and Maximal Congruence in Transformation Semigroups with Fixed Sets

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Abstract : An element a of a semigroup S is called left (right) regular if there exists x in S such that $a=xa^2$ ($a=a^2x$) and said to be intra-regular if there exist u,v in S such that $a=ua^2v$. Let $T(X)$ be the semigroup of all full transformations on a set X under the composition of maps. For a fixed nonempty subset Y of X , let $\text{Fix}(X,Y)=\{\alpha \in T(X) : y\alpha=y \text{ for all } y \in Y\}$, where $y\alpha$ is the image of y under α . Then $\text{Fix}(X,Y)$ is a semigroup of full transformations on X which fix all elements in Y . Here, we characterize left regular, right regular and intra-regular elements of $\text{Fix}(X,Y)$ which characterizations are shown as follows: For $\alpha \in \text{Fix}(X,Y)$, (i) α is left regular if and only if $X\alpha \setminus Y = X\alpha^2 \setminus Y$, (ii) α is right regular if and only if $\pi\alpha = \pi\alpha^2$, (iii) α is intra-regular if and only if $|X\alpha \setminus Y| = |X\alpha^2 \setminus Y|$ such that $X\alpha = \{x\alpha : x \in X\}$ and $\pi\alpha = \{x\alpha^{-1} : x \in X\}$ in which $x\alpha^{-1} = \{a \in X : a\alpha=x\}$. Moreover, those regularities are equivalent if $X\alpha \setminus Y$ is a finite set. In addition, we count the number of those elements of $\text{Fix}(X,Y)$ when X is a finite set. Finally, we determine the maximal congruence ρ on $\text{Fix}(X,Y)$ when X is finite and Y is a nonempty proper subset of X . If we let $|X \setminus Y| = n$, then we obtain that $\rho = (\text{Fix}_n \times \text{Fix}_n) \cup (H \varepsilon \times H \varepsilon)$ where $\text{Fix}_n = \{\alpha \in \text{Fix}(X,Y) : |X\alpha \setminus Y| < n\}$ and $H \varepsilon$ is the group of units of $\text{Fix}(X,Y)$. Furthermore, we show that the maximal congruence is unique.

Keywords : intra-regular, left regular, maximal congruence, right regular, transformation semigroup

Conference Title : ICAPM 2017 : International Conference on Applied Physics and Mathematics

Conference Location : Prague, Czechia

Conference Dates : March 23-24, 2017