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Regularity and Maximal Congruence in Transformation Semigroups with Fixed Sets

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Abstract : An element a of a semigroup S is called left (right) regular if there exists x in S such that $a=xa^2$ ($a=a^2x$) and said to be intra-regular if there exist u,v in such that $a=ua^2v$. Let T(X) be the semigroup of all full transformations on a set X under the composition of maps. For a fixed nonempty subset Y of X, let Fix(X,Y)={α ™ T(X) : yα=y for all y ™ Y}, where yα is the image of y under α. Then Fix(X,Y) is a semigroup of full transformations on X which fix all elements in Y. Here, we characterize left regular, right regular and intra-regular elements of Fix(X,Y) which characterizations are shown as follows: For α ™ Fix(X,Y), (i) α is left regular if and only if Xα\Y = $X\alpha^2$\Y, (ii) α is right regular if and only if πα = $\pi\alpha^2$, (iii) α is intra-regular if and only if | Xα\Y | = | $X\alpha^2$\Y | such that Xα = {xα : x ™ X} and πα = {$x\alpha^{-1}$: x ™ Xα} in which $x\alpha^{-1}$ = {a ™ X : aα=x}. Moreover, those regularities are equivalent if Xα\Y is a finite set. In addition, we count the number of those elements of Fix(X,Y) when X is a finite set. Finally, we determine the maximal congruence ρ on Fix(X,Y) when X is finite and Y is a nonempty proper subset of X. If we let | X \Y | = n, then we obtain that ρ = (Fixn x Fixn) ∪ (H ε x H ε) where Fixn = {α ™ Fix(X,Y) : | Xα\Y | < n} and H ε is the group of units of Fix(X,Y). Furthermore, we show that the maximal congruence is unique.

Keywords : intra-regular, left regular, maximal congruence, right regular, transformation semigroup
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