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Semigroups of Linear Transformations with Fixed Subspaces: Green's Relations and Ideals

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Abstract : Let V be a vector space over a field and W a subspace of V. Let Fix(V,W) denote the set of all linear transformations on V with fix all elements in W. In this paper, we show that Fix(V,W) is a semigroup under the composition of maps and describe Green's relations on this semigroup in terms of images, kernels and the dimensions of subspaces of the quotient space V/W where V/W = {v+W : v is an element in V} with v+W = {v+w : w is an element in W}. Let dim(U) denote the dimension of a vector space U and Vα = {vα : v is an element in V} where vα is an image of v under a linear transformation α. For any cardinal number a let a'= min{b : b > a}. We also show that the ideals of Fix(V,W) are precisely the sets. Fix(r) ={α Fix(V,W) : dim(Vα/W) < r} where 1 ≤ r ≤ a' and a = dim(V/W). Moreover, we prove that if V is a finite-dimensional vector space, then every ideal of Fix(V,W) is principle.

Keywords : Green's relations, ideals, linear transformation semi-groups, principle ideals
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