

## Semigroups of Linear Transformations with Fixed Subspaces: Green's Relations and Ideals

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**Abstract :** Let  $V$  be a vector space over a field and  $W$  a subspace of  $V$ . Let  $\text{Fix}(V,W)$  denote the set of all linear transformations on  $V$  with fix all elements in  $W$ . In this paper, we show that  $\text{Fix}(V,W)$  is a semigroup under the composition of maps and describe Green's relations on this semigroup in terms of images, kernels and the dimensions of subspaces of the quotient space  $V/W$  where  $V/W = \{v+W : v \text{ is an element in } V\}$  with  $v+W = \{v+w : w \text{ is an element in } W\}$ . Let  $\dim(U)$  denote the dimension of a vector space  $U$  and  $V\alpha = \{v\alpha : v \text{ is an element in } V\}$  where  $v\alpha$  is an image of  $v$  under a linear transformation  $\alpha$ . For any cardinal number  $a$  let  $a' = \min\{b : b > a\}$ . We also show that the ideals of  $\text{Fix}(V,W)$  are precisely the sets.  $\text{Fix}(r) = \{\alpha \in \text{Fix}(V,W) : \dim(V\alpha/W) < r\}$  where  $1 \leq r \leq a'$  and  $a = \dim(V/W)$ . Moreover, we prove that if  $V$  is a finite-dimensional vector space, then every ideal of  $\text{Fix}(V,W)$  is principle.

**Keywords :** Green's relations, ideals, linear transformation semi-groups, principle ideals

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