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[Keynote Talk]: Discovering Liouville-Type Problems for p-Energy Minimizing Maps in Closed Half-Ellipsoids by Calculus Variation Method

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Abstract : The goal of this project is to investigate constant properties (called the Liouville-type Problem) for a p-stable map as a local or global minimum of a p-energy functional where the domain is a Euclidean space and the target space is a closed half-ellipsoid. The First and Second Variation Formulas for a p-energy functional has been applied in the Calculus Variation Method as computation techniques. Stokes' Theorem, Cauchy-Schwarz Inequality, Hardy-Sobolev type Inequalities, and the Bochner Formula as estimation techniques have been used to estimate the lower bound and the upper bound of the derived p-Harmonic Stability Inequality. One challenging point in this project is to construct a family of variation maps such that the images of variation maps must be guaranteed in a closed half-ellipsoid. The other challenging point is to find a contradiction between the lower bound and the upper bound in an analysis of p-Harmonic Stability Inequality when a p-energy minimizing map is not constant. Therefore, the possibility of a non-constant p-energy minimizing map has been ruled out and the constant property for a p-energy minimizing map has been obtained. Our research finding is to explore the constant property for a p-stable map from a Euclidean space into a closed half-ellipsoid in a certain range of p. The certain range of p is determined by the dimension values of a Euclidean space (the domain) and an ellipsoid (the target space). The certain range of p is also bounded by the curvature values on an ellipsoid (that is, the ratio of the longest axis to the shortest axis). Regarding Liouville-type results for a p-stable map, our research finding on an ellipsoid is a generalization of mathematicians' results on a sphere. Our result is also an extension of mathematicians' Liouville-type results from a special ellipsoid with only one parameter to any ellipsoid with (n+1) parameters in the general setting.

Keywords : Bochner formula, Calculus Stokes' Theorem, Cauchy-Schwarz Inequality, first and second variation formulas, Liouville-type problem, p-harmonic map
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