World Academy of Science, Engineering and Technology
International Journal of Mathematical and Computational Sciences
Vol:10, No:08, 2016

Periodicity of Solutions to Impulsive Equations

Authors : Jin Liang, James H. Liu, Ti-Jun Xiao

Abstract : It is known that there exist many physical phenomena where abrupt or impulsive changes occur either in the system dynamics, for example, ad-hoc network, or in the input forces containing impacts, for example, the bombardment of space antenna by micrometeorites. There are many other examples such as ultra high-speed optical signals over communication networks, the collision of particles, inventory control, government decisions, interest changes, changes in stock price, etc. These are impulsive phenomena. Hence, as a combination of the traditional initial value problems and the short-term perturbations whose duration can be negligible in comparison with the duration of the process, the systems with impulsive conditions (i.e., impulsive systems) are more realistic models for describing the impulsive phenomenon. Such a situation is also suitable for the delay systems, which include some of the past states of the system. So far, there have been a lot of research results in the study of impulsive systems with delay both in finite and infinite dimensional spaces. In this paper, we investigate the periodicity of solutions to the nonautonomous impulsive evolution equations with infinite delay in Banach spaces, where the coefficient operators (possibly unbounded) in the linear part depend on the time, which are impulsive systems in infinite dimensional spaces and come from the optimal control theory. It was indicated that the study of periodic solutions for these impulsive evolution equations with infinite delay was challenging because the fixed point theorems requiring some compactness conditions are not applicable to them due to the impulsive condition and the infinite delay. We are happy to report that after detailed analysis, we are able to combine the techniques developed in our previous papers, and some new ideas in this paper, to attack these impulsive evolution equations and derive periodic solutions. More specifically, by virtue of the related transition operator family (evolution family), we present a Poincaré operator given by the nonautonomous impulsive evolution system with infinite delay, and then show that the operator is a condensing operator with respect to Kuratowski's measure of non-compactness in a phase space by using an Amann's lemma. Finally, we derive periodic solutions from bounded solutions in view of the Sadovskii fixed point theorem. We also present a relationship between the boundedness and the periodicity of the solutions of the nonautonomous impulsive evolution system. The new results obtained here extend some earlier results in this area for evolution equations without impulsive conditions or without infinite delay.

Keywords : impulsive, nonautonomous evolution equation, optimal control, periodic solution
Conference Title : ICMCS 2016 : International Conference on Mathematics, Computics and Statistics
Conference Location : Seattle, United States
Conference Dates : August 08-09, 2016