Fractal Analysis of Some Bifurcations of Discrete Dynamical Systems in Higher Dimensions

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Abstract : The main purpose of this paper is to study the box dimension as fractal property of bifurcations of discrete dynamical systems in higher dimensions. The paper contains the fractal analysis of the orbits near the hyperbolic and non-hyperbolic fixed points in discrete dynamical systems. It is already known that in one-dimensional case the orbit near the hyperbolic fixed point has the box dimension equal to zero. On the other hand, the orbit near the non-hyperbolic fixed point has strictly positive box dimension which is connected to the non-degeneracy condition of certain bifurcation. One of the main results in this paper is the generalisation of results about box dimension near the hyperbolic and non-hyperbolic fixed points to higher dimensions. In the process of determining box dimension, the restriction of systems to stable, unstable and center manifolds, Lipschitz property of box dimension and the notion of projective box dimension are used. The analysis of the bifurcations in higher dimensions with one multiplier on the unit circle is done by using the normal forms on one-dimensional center manifolds. This specific change in box dimension of an orbit at the moment of bifurcation has already been explored for some bifurcations in one and two dimensions. It was shown that specific values of box dimension are connected to appropriate bifurcations such as fold, flip, cusp or Neimark-Sacker bifurcation. This paper further explores this connection of box dimension as fractal property to some specific bifurcations in higher dimensions, such as fold-flip and flip-Neimark-Sacker. Furthermore, the application of the results to the unit time map of continuous dynamical system near hyperbolic and non-hyperbolic singularities is presented. In that way, box dimensions which are specific for certain bifurcations of continuous systems can be obtained. The approach to bifurcation analysis by using the box dimension as specific fractal property of orbits can lead to better understanding of bifurcation phenomenon. It could also be useful in detecting the existence or nonexistence of bifurcations of discrete and continuous dynamical systems.

Keywords : bifurcation, box dimension, invariant manifold, orbit near fixed point

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