

## **A Variational Reformulation for the Thermomechanically Coupled Behavior of Shape Memory Alloys**

**Authors :** Elisa Boatti, Ulisse Stefanelli, Alessandro Reali, Ferdinando Auricchio

**Abstract :** Thanks to their unusual properties, shape memory alloys (SMAs) are good candidates for advanced applications in a wide range of engineering fields, such as automotive, robotics, civil, biomedical, aerospace. In the last decades, the ever-growing interest for such materials has boosted several research studies aimed at modeling their complex nonlinear behavior in an effective and robust way. Since the constitutive response of SMAs is strongly thermomechanically coupled, the investigation of the non-isothermal evolution of the material must be taken into consideration. The present study considers an existing three-dimensional phenomenological model for SMAs, able to reproduce the main SMA properties while maintaining a simple user-friendly structure, and proposes a variational reformulation of the full non-isothermal version of the model. While the considered model has been thoroughly assessed in an isothermal setting, the proposed formulation allows to take into account the full nonisothermal problem. In particular, the reformulation is inspired to the GENERIC (General Equations for Non-Equilibrium Reversible-Irreversible Coupling) formalism, and is based on a generalized gradient flow of the total entropy, related to thermal and mechanical variables. Such phrasing of the model is new and allows for a discussion of the model from both a theoretical and a numerical point of view. Moreover, it directly implies the dissipativity of the flow. A semi-implicit time-discrete scheme is also presented for the fully coupled thermomechanical system, and is proven unconditionally stable and convergent. The correspondent algorithm is then implemented, under a space-homogeneous temperature field assumption, and tested under different conditions. The core of the algorithm is composed of a mechanical subproblem and a thermal subproblem. The iterative scheme is solved by a generalized Newton method. Numerous uniaxial and biaxial tests are reported to assess the performance of the model and algorithm, including variable imposed strain, strain rate, heat exchange properties, and external temperature. In particular, the heat exchange with the environment is the only source of rate-dependency in the model. The reported curves clearly display the interdependence between phase transformation strain and material temperature. The full thermomechanical coupling allows to reproduce the exothermic and endothermic effects during respectively forward and backward phase transformation. The numerical tests have thus demonstrated that the model can appropriately reproduce the coupled SMA behavior in different loading conditions and rates. Moreover, the algorithm has proved effective and robust. Further developments are being considered, such as the extension of the formulation to the finite-strain setting and the study of the boundary value problem.

**Keywords :** generalized gradient flow, GENERIC formalism, shape memory alloys, thermomechanical coupling

**Conference Title :** ICCM 2016 : International Conference on Computational Mechanics

**Conference Location :** Venice, Italy

**Conference Dates :** August 08-09, 2016