

First Order Moment Bounds on DMRL and IMRL Classes of Life Distributions

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Abstract : The class of life distributions with decreasing mean residual life (DMRL) is well known in the field of reliability modeling. It contains the IFR class of distributions and is contained in the NBUE class of distributions. While upper and lower bounds of the reliability distribution function of aging classes such as IFR, IFRA, NBU, NBUE, and HNBUE have discussed in the literature for a long time, there is no analogous result available for the DMRL class. We obtain the upper and lower bounds for the reliability function of the DMRL class in terms of first order finite moment. The lower bound is obtained by showing that for any fixed time, the minimization of the reliability function over the class of all DMRL distributions with a fixed mean is equivalent to its minimization over a smaller class of distribution with a special form. Optimization over this restricted set can be made algebraically. Likewise, the maximization of the reliability function over the class of all DMRL distributions with a fixed mean turns out to be a parametric optimization problem over the class of DMRL distributions of a special form. The constructive proofs also establish that both the upper and lower bounds are sharp. Further, the DMRL upper bound coincides with the HNBUE upper bound and the lower bound coincides with the IFR lower bound. We also prove that a pair of sharp upper and lower bounds for the reliability function when the distribution is increasing mean residual life (IMRL) with a fixed mean. This result is proved in a similar way. These inequalities fill a long-standing void in the literature of the life distribution modeling.

Keywords : DMRL, IMRL, reliability bounds, hazard functions

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