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Combining a Continuum of Hidden Regimes and a Heteroskedastic Three-Factor Model in Option Pricing

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Abstract : This paper develops a discrete-time option pricing model for index options. The model consists of two key ingredients. First, daily stock return innovations are driven by a continuous hidden threshold mixed skew-normal (HTSN) distribution which generates conditional non-normality that is needed to fit daily index return. The most important feature of the HTSN is the inclusion of a latent state variable with a continuum of states, unlike the traditional mixture distributions where the state variable is discrete with little number of states. The HTSN distribution belongs to the class of univariate probability distributions where parameters of the distribution capture the dependence between the variable of interest and the continuous latent state variable (the regime). The distribution has an interpretation in terms of a mixture distribution with time-varying mixing probabilities. It has been shown empirically that this distribution outperforms its main competitor, the mixed normal (MN) distribution, in terms of capturing the stylized facts known for stock returns, namely, volatility clustering, leverage effect, skewness, kurtosis and regime dependence. Second, heteroscedasticity in the model is captured by a threeexogenous-factor GARCH model (GARCHX), where the factors are taken from the principal components analysis of various world indices and presents an application to option pricing. The factors of the GARCHX model are extracted from a matrix of world indices applying principal component analysis (PCA). The empirically determined factors are uncorrelated and represent truly different common components driving the returns. Both factors and the eight parameters inherent to the HTSN distribution aim at capturing the impact of the state of the economy on price levels since distribution parameters have economic interpretations in terms of conditional volatilities and correlations of the returns with the hidden continuous state. The PCA identifies statistically independent factors affecting the random evolution of a given pool of assets -in our paper a pool of international stock indices- and sorting them by order of relative importance. The PCA computes a historical cross asset covariance matrix and identifies principal components representing independent factors. In our paper, factors are used to calibrate the HTSN-GARCHX model and are ultimately responsible for the nature of the distribution of random variables being generated. We benchmark our model to the MN-GARCHX model following the same PCA methodology and the standard Black-Scholes model. We show that our model outperforms the benchmark in terms of RMSE in dollar losses for put and call options, which in turn outperforms the analytical Black-Scholes by capturing the stylized facts known for index returns, namely, volatility clustering, leverage effect, skewness, kurtosis and regime dependence.

Keywords : continuous hidden threshold, factor models, GARCHX models, option pricing, risk-premium
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