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Robust Inference with a Skew T Distribution

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Abstract : There is a growing body of evidence that non-normal data is more prevalent in nature than the normal one. Examples can be quoted from, but not restricted to, the areas of Economics, Finance and Actuarial Science. The non-normality considered here is expressed in terms of fat-tailedness and asymmetry of the relevant distribution. In this study a skew t distribution that can be used to model a data that exhibit inherent non-normal behavior is considered. This distribution has tails fatter than a normal distribution and it also exhibits skewness. Although maximum likelihood estimates can be obtained by solving iteratively the likelihood equations that are non-linear in form, this can be problematic in terms of convergence and in many other respects as well. Therefore, it is preferred to use the method of modified maximum likelihood in which the likelihood estimates are derived by expressing the intractable non-linear likelihood equations in terms of standardized ordered variates and replacing the intractable terms by their linear approximations obtained from the first two terms of a Taylor series expansion about the quantiles of the distribution. These estimates, called modified maximum likelihood estimates, are obtained in closed form. Hence, they are easy to compute and to manipulate analytically. In fact the modified maximum likelihood estimates are equivalent to maximum likelihood estimates, asymptotically. Even in small samples the modified maximum likelihood estimates are found to be approximately the same as maximum likelihood estimates that are obtained iteratively. It is shown in this study that the modified maximum likelihood estimates are not only unbiased but substantially more efficient than the commonly used moment estimates or the least square estimates that are known to be biased and inefficient in such cases. Furthermore, in conventional regression analysis, it is assumed that the error terms are distributed normally and, hence, the well-known least square method is considered to be a suitable and preferred method for making the relevant statistical inferences. However, a number of empirical researches have shown that non-normal errors are more prevalent. Even transforming and/or filtering techniques may not produce normally distributed residuals. Here, a study is done for multiple linear regression models with random error having non-normal pattern. Through an extensive simulation it is shown that the modified maximum likelihood estimates of regression parameters are plausibly robust to the distributional assumptions and to various data anomalies as compared to the widely used least square estimates. Relevant tests of hypothesis are developed and are explored for desirable properties in terms of their size and power. The tests based upon modified maximum likelihood estimates are found to be substantially more powerful than the tests based upon least square estimates. Several examples are provided from the areas of Economics and Finance where such distributions are interpretable in terms of efficient market hypothesis with respect to asset pricing, portfolio selection, risk measurement and capital allocation, etc.

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