

Optimal Perturbation in an Impulsively Blocked Channel Flow

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Abstract : The current work implements the variational principle to find the optimum initial perturbation that provides maximum growth in an impulsively blocked channel flow. The conventional method for studying temporal stability has always been through modal analysis. In most of the transient flows, this modal analysis is still followed with the quasi-steady assumption, i.e. change in base flow is much slower compared to perturbation growth rate. There are other studies where transient analysis on time dependent flows is done by formulating the growth of perturbation as an initial value problem. But the perturbation growth is sensitive to the initial condition. This study intends to find the initial perturbation that provides the maximum growth at a later time. Here, the expression of base flow for blocked channel is derived and the formulation is based on the two dimensional perturbation with stream function representing the perturbation quantity. Hence, the governing equation becomes the Orr-Sommerfeld equation. In the current context, the cost functional is defined as the ratio of disturbance energy at a terminal time 'T' to the initial energy, i.e. $G(T) = ||q(T)||^2 / ||q(0)||^2$ where q is the perturbation and $||\cdot||$ defines the norm chosen. The above cost functional needs to be maximized against the initial perturbation distribution. It is achieved with the constraint that perturbation follows the basic governing equation, i.e. Orr-Sommerfeld equation. The corresponding adjoint equation is derived and is solved along with the basic governing equation in an iterative manner to provide the initial spatial shape of the perturbation that provides the maximum growth $G(T)$. The growth rate is plotted against time showing the development of perturbation which achieves an asymptotic shape. The effects of various parameters, e.g. Reynolds number, are studied in the process. Thus, the study emphasizes on the usage of optimal perturbation and its growth to understand the stability characteristics of time dependent flows. The assumption of quasi-steady analysis can be verified against these results for the transient flows like impulsive blocked channel flow.

Keywords : blocked channel flow, calculus of variation, hydrodynamic stability, optimal perturbation

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