On the Basis Number and the Minimum Cycle Bases of the Wreath Product of Paths with Wheels

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Abstract : For a given graph G, the set \mathcal{E} of all subsets of E(G) forms an |E(G)| dimensional vector space over Z2 with vector addition $X \oplus Y = (X \setminus Y)$ [(Y \X) and scalar multiplication 1.X = X and $0.X = \emptyset$ for all X, Ye \mathcal{E} . The cycle space, C(G), of a graph G is the vector subspace of (E; \oplus ; .) spanned by the cycles of G. Traditionally there have been two notions of minimality among bases of C(G). First, a basis B of G is called a d-fold if each edge of G occurs in at most d cycles of the basis B. The basis number, b(G), of G is the least non-negative integer d such that C(G) has a d-fold basis; a required basis of C(G) is a basis for which each edge of G belongs to at most b(G) elements of B. Second, a basis B is called a minimum cycle basis (MCB) if its total length Σ BeB |B| is minimum among all bases of C(G). The lexicographic product GpH has the vertex set V (GpH) = V (G) x V (H) and the edge set E(GpH) = {(u1, v1)(u2, v2)|u1 = u2 and v1 v2 \in E(H); or u1u2 \in E(G) and there is $\alpha \in$ Aut(H) such that α (v1) = v2}. In this work, a construction of a minimum cycle basis for the wreath product of wheels with paths is presented. Also, the length of the longest cycle of a minimum cycle basis is determined. Moreover, the basis number for the wreath product of the same is investigated.

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Keywords : cycle space, minimum cycle basis, basis number, wreath product

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