

## On the Basis Number and the Minimum Cycle Bases of the Wreath Product of Paths with Wheels

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**Abstract :** For a given graph  $G$ , the set  $\mathcal{E}$  of all subsets of  $E(G)$  forms an  $|E(G)|$  dimensional vector space over  $Z_2$  with vector addition  $X \oplus Y = (X \setminus Y) \cup (Y \setminus X)$  and scalar multiplication  $1.X = X$  and  $0.X = \emptyset$  for all  $X, Y \in \mathcal{E}$ . The cycle space,  $C(G)$ , of a graph  $G$  is the vector subspace of  $(\mathcal{E}; \oplus; \cdot)$  spanned by the cycles of  $G$ . Traditionally there have been two notions of minimality among bases of  $C(G)$ . First, a basis  $B$  of  $G$  is called a  $d$ -fold if each edge of  $G$  occurs in at most  $d$  cycles of the basis  $B$ . The basis number,  $b(G)$ , of  $G$  is the least non-negative integer  $d$  such that  $C(G)$  has a  $d$ -fold basis; a required basis of  $C(G)$  is a basis for which each edge of  $G$  belongs to at most  $b(G)$  elements of  $B$ . Second, a basis  $B$  is called a minimum cycle basis (MCB) if its total length  $\sum_{B \in \mathcal{B}} |B|$  is minimum among all bases of  $C(G)$ . The lexicographic product  $G \rho H$  has the vertex set  $V(G \rho H) = V(G) \times V(H)$  and the edge set  $E(G \rho H) = \{(u_1, v_1)(u_2, v_2) | u_1 = u_2 \text{ and } v_1 v_2 \in E(H); \text{ or } u_1 u_2 \in E(G) \text{ and there is } \alpha \in \text{Aut}(H) \text{ such that } \alpha(v_1) = v_2\}$ . In this work, a construction of a minimum cycle basis for the wreath product of wheels with paths is presented. Also, the length of the longest cycle of a minimum cycle basis is determined. Moreover, the basis number for the wreath product of the same is investigated.

**Keywords :** cycle space, minimum cycle basis, basis number, wreath product

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