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A New Formulation Of The M And M-theta Integrals Generalized For Virtual Crack Closure In A Three-dimensional Medium

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Abstract : The safety and durability of structures remain challenging fields that continue to draw the attention of designers. One widely adopted approach is fracture mechanics, which provides methods to evaluate crack stability in complex geometries and under diverse loading conditions. The global energy approach is particularly comprehensive, as it calculates the energy release rate required for crack initiation and propagation using path-independent integrals. This study aims to extend these invariant integrals to include path-independent integrals, with the goal of enhancing the accuracy of failure predictions. The ultimate objective is to create more robust materials while optimizing structural safety and durability. By integrating the real and virtual field method with the virtual crack closure technique, a new formulation of the M-integral is introduced. This formulation establishes a direct relationship between local stresses on the crack faces and the opening displacements, allowing for an accurate calculation of fracture energy. The analytical calculations are grounded in the assumption that the energy needed to close a crack virtually is equal to the energy released during its opening. This novel integral is implemented in a finite element code using Cast3M to simulate cracking criteria within a wood material context. Initially, the numerical calculations are focused on plane strain conditions, but they are later extended to three-dimensional environments, taking into account the orthotropic nature of wood.

Keywords : energy release rate, path-independent integrals, virtual crack closure, orthotropic material
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