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Prime Graphs of Polynomials and Power Series Over Non-Commutative Rings

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Abstract : Algebraic graph theory is defined as a bridge between algebraic structures and graphs. It has several uses in many fields, including chemistry, physics, and computer science. The prime graph is a type of graph associated with a ring R, where the vertex set is the whole ring R, and two vertices x and y are adjacent if either xRy=0 or yRx=0. However, the investigation of the prime graph over rings remains relatively limited. The behavior of this graph in extended rings, like R[x] and R[[x]], where R is a non-commutative ring, deserves more attention because of the wider applicability in algebra and other mathematical fields. To study the prime graphs over polynomials and power series rings, we used a combination of ring-theoretic and graph-theoretic techniques. This paper focuses on two invariants: the diameter and the girth of these graphs. Furthermore, the work discusses how the graph structures change when passing from R to R[x] and R[[x]]. In our study, we found that the set of strong zero-divisors of ring R represents the set of vertices in prime graphs. Based on this discovery, we redefined the vertices of prime graphs using the definition of strong zero divisors. Additionally, our results show that although the prime graphs of R[x] and R[[x]] are comparable to the graph of R, they have different combinatorial characteristics since these extensions contain new strong zero-divisors. In particular, we find conditions in which the diameter and girth of the graphs, as they expand from R to R[x] and R[[x]], do not change or do change. In conclusion, this study shows how extending a non-commutative ring R to R[x] and R[[x]] affects the structure of their prime graphs, particularly in terms of diameter and girth. These findings enhance the understanding of the relationship between ring extensions and graph properties.

Keywords : prime graph, diameter, girth, polynomial ring, power series ring
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