World Academy of Science, Engineering and Technology
International Journal of Mathematical and Computational Sciences
Vol:19, No:02, 2025

Enhanced Tensor Tomographic Reconstruction: Integrating Absorption, Refraction and Temporal Effects

Authors : Lukas Vierus, Thomas Schuster

Abstract : A general framework is examined for dynamic tensor field tomography within an inhomogeneous medium characterized by refraction and absorption, treated as an inverse source problem concerning the associated transport equation. Guided by Fermat’s principle, the Riemannian metric within the specified domain is determined by the medium's refractive index. While considerable literature exists on the inverse problem of reconstructing a tensor field from its longitudinal ray transform within a static Euclidean environment, limited inversion formulas and algorithms are available for general Riemannian metrics and time-varying tensor fields. It is established that tensor field tomography, akin to an inverse source problem for a transport equation, persists in dynamic scenarios. Framing dynamic tensor tomography as an inverse source problem embodies a comprehensive perspective within this domain. Ensuring well-defined forward mappings necessitates establishing existence and uniqueness for the underlying transport equations. However, the bilinear forms of the associated weak formulations fail to meet the coercivity condition. Consequently, recourse to viscosity solutions is taken, demonstrating their unique existence within suitable Sobolev spaces (in the static case) and Sobolev-Bochner spaces (in the dynamic case), under a specific assumption restricting variations in the refractive index. Notably, the adjoint problem can also be reformulated as a transport equation, with analogous results regarding uniqueness. Analytical solutions are expressed as integrals over geodesics, facilitating more efficient evaluation of forward and adjoint operators compared to solving partial differential equations. Certainly, here's the revised sentence in English: Numerical experiments are conducted using a Nesterov-accelerated Landweber method, encompassing various fields, absorption coefficients, and refractive indices, thereby illustrating the enhanced reconstruction achieved through this holistic modeling approach.

Keywords : attenuated refractive dynamic ray transform of tensor fields, geodesics, transport equation, viscosity solutions
Conference Title : ICIPE 2025 : International Conference on Inverse Problems in Engineering
Conference Location : Lisbon, Portugal
Conference Dates : February 08-09, 2025