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Pressure-Robust Approximation for the Rotational Fluid Flow Problems

Authors : Medine Demir, Volker John

Abstract : Fluid equations in a rotating frame of reference have a broad class of important applications in meteorology and oceanography, especially in the large-scale flows considered in ocean and atmosphere, as well as many physical and industrial applications. The Coriolis and the centripetal forces, resulting from the rotation of the earth, play a crucial role in such systems. For such applications it may be required to solve the system in complex three-dimensional geometries. In recent years, the Navier--Stokes equations in a rotating frame have been investigated in a number of papers using the classical inf-sup stable mixed methods, like Taylor-Hood pairs, to contribute to the analysis and the accurate and efficient numerical simulation. Numerical analysis reveals that these classical methods introduce a pressure-dependent contribution in the velocity error bounds that is proportional to some inverse power of the viscosity. Hence, these methods are optimally convergent but small velocity errors might not be achieved for complicated pressures and small viscosity coefficients. Several approaches have been proposed for improving the pressure-robustness of pairs of finite element spaces. In this contribution, a pressure-robust space discretization of the incompressible Navier--Stokes equations in a rotating frame of reference is considered. The discretization employs divergence-free, H^1-conforming mixed finite element methods like Scott--Vogelius pairs. However, this approach might come with a modification of the meshes, like the use of barycentric-refined grids in case of Scott--Vogelius pairs. However, this strategy requires the finite element code to have control on the mesh generator which is not realistic in many engineering applications and might also be in conflict with the solver for the linear system. An error estimate for the velocity is derived that tracks the dependency of the error bound on the coefficients of the problem, in particular on the angular velocity. Numerical examples illustrate the theoretical results. The idea of pressure-robust method could be cast on different types of flow problems which would be considered as future studies. As another future research direction, to avoid a modification of the mesh, one may use a very simple parameter-dependent modification of the Scott-Vogelius element, the pressure-wired Stokes element, such that the inf-sup constant is independent of nearly-singular vertices.

Keywords : navier-stokes equations in a rotating frame of refence, coriolis force, pressure-robust error estimate, scott-vogelius pairs of finite element spaces
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