Generalization of Tsallis Entropy from a Q-Deformed Arithmetic

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Abstract : It is known that by introducing alternative forms of exponential and logarithmic functions, the Tsallis entropy Sq is itself a generalization of Shannon entropy S. In this work, from a deformation through a scaling function applied to the differential operator, it is possible to generate a q-deformed calculus as well as a q-deformed arithmetic, which not only allows generalizing the exponential and logarithmic functions but also any other standard function. The updated q-deformed differential operator leads to an updated integral operator under which the functions are integrated together with a weight function. For each differentiable function, it is possible to identify its q-deformed partner, which is useful to generalize other algebraic relations proper of the original functions. As an application of this proposal, in this work, a generalization of exponential and logarithmic functions is studied in such a way that their relationship with the thermodynamic functions, particularly the entropy, allows us to have a q-deformed expression of these. As a result, from a particular scaling function applied to the differential operator, a q-deformed arithmetic is obtained, leading to the generalization of the Tsallis entropy.

 ${\bf Keywords: q-calculus, q-deformed \ arithmetic, \ entropy, \ exponential \ functions, \ thermodynamic \ functions}$

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