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Bound State Problems and Functional Differential Geometry

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Abstract : We study a class of functional partial differential equations(FPDEs). This class is suggested by Quantum Field Theory. We derive general properties of solutions to such equations. In particular, we demonstrate that they lead to systems of coupled integral equations with singular kernels. We show that solutions to such hierarchies can be sought among functions with regular singularities at a countable set of subvarieties of the physical space. We also develop a formal analogy of basic constructions of differential geometry on functional manifolds, as this is necessary for in depth study of FPDEs. We also consider the case of linear overdetermined systems of functional differential equations and show that it can be completely solved in terms of formal solutions of a functional equation that is a functional analogy of a system of determined algebraic equations. This development leads us to formally define the functional analogy of algebraic geometry, which we call functional algebraic geometry. We study basic properties of functional algebraic varieties. In particular, we investigate the case of a formally discrete set of solutions. We also define and study functional analogy of discriminants. In the case of fully determined systems such that the defining functionals have regular singularities, we demonstrate that formal solutions can be sought in the class of functions with regular singularities. This case provides a practical way to apply our results to physics problems.

Keywords : functional equations, quantum field theory, holomorphic functions, Yang Mills mass gap problem, quantum chaos
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