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New Hardy Type Inequalities of Two-Dimensional on Time Scales via Steklov Operator

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Abstract : The mathematical inequalities have been the core of mathematical study and used in almost all branches of mathematics as well in various areas of science and engineering. The inequalities by Hardy, Littlewood and Polya were the first significant composition of several science. This work presents fundamental ideas, results and techniques, and it has had much influence on research in various branches of analysis. Since 1934, various inequalities have been produced and studied in the literature. Furthermore, some inequalities have been formulated by some operators; in 1989, weighted Hardy inequalities have been obtained for integration operators. Then, they obtained weighted estimates for Steklov operators that were used in the solution of the Cauchy problem for the wave equation. They were improved upon in 2011 to include the boundedness of integral operators from the weighted Sobolev space to the weighted Lebesgue space. Some inequalities have been demonstrated and improved using the Hardy–Steklov operator. Recently, a lot of integral inequalities have been improved by differential operators. Hardy inequality has been one of the tools that is used to consider integrity solutions of differential equations. Then, dynamic inequalities of Hardy and Coposon have been extended and improved by various integral operators. These inequalities would be interesting to apply in different fields of mathematics (functional spaces, partial differential equations, mathematical modeling). Some inequalities have been appeared involving Copson and Hardy inequalities on time scales to obtain new special version of them. A time scale is an arbitrary nonempty closed subset of the real numbers. Then, the dynamic inequalities on time scales have received a lot of attention in the literature and has become a major field in pure and applied mathematics. There are many applications of dynamic equations on time scales to quantum mechanics, electrical engineering, neural networks, heat transfer, combinatorics, and population dynamics. This study focuses on Hardy and Coposon inequalities, using Steklov operator on time scale in double integrals to obtain special cases of time-scale inequalities of Hardy and Copson on high dimensions. The advantage of this study is that it uses the one-dimensional classical Hardy inequality to obtain higher dimensional on time scale versions that will be applied in the solution of the Cauchy problem for the wave equation. In addition, the obtained inequalities have various applications involving discontinuous domains such as bug populations, phytoremediation of metals, wound healing, maximization problems. The proof can be done by introducing restriction on the operator in several cases. The concepts in time scale version such as time scales calculus will be used that allows to unify and extend many problems from the theories of differential and of difference equations. In addition, using chain rule, and some properties of multiple integrals on time scales, some theorems of Fubini and the inequality of H¨older.

Keywords : time scales, inequality of hardy, inequality of coposon, steklov operator
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