World Academy of Science, Engineering and Technology
International Journal of Mathematical and Computational Sciences
Vol:17, No:12, 2023

On Lie Groupoids, Bundles, and Their Categories

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Abstract : A Lie group is a highly sophisticated structure which is a smooth manifold whose underlying set of elements is equipped with the structure of a group such that the group multiplication and inverse-assigning functions are smooth. This structure was introduced by the Norwegian mathematician So- phus Lie who founded the theory of continuous groups. The Lie groups are well developed and have wide applications in areas including Mathematical Physics. There are several advances and generalizations for Lie groups and Lie groupoids is one such which is termed as a "many-object generalization" of Lie groups. A groupoid is a category whose morphisms are all invertible, obviously, every group is a groupoid but not conversely. Definition 1. A Lie groupoid G ⇒ M is a groupoid G on a base M together with smooth structures on G and M such that the maps α, β: G → M are surjective submertions, the object inclusion map x '→ 1x, M → G is smooth, and the partial multiplication G ∗ G → G is smooth. A bundle is a triple (E, p, B) where E, B are topological spaces p: E → B is a map. Space B is called the base space and space E is called total space and map p is the projection of the bundle. For each b ∈ B, the space p−1(b) is called the fibre of the bundle over b ∈ B. Intuitively a bundle is regarded as a union of fibres p−1(b) for b ∈ B parametrized by B and 'glued together' by the topology of the space E. A cross-section of a bundle (E, p, B) is a map s: B → E such that ps = 1B. Example 1. Given any space B, a product bundle over B with fibre F is (B × F, p, B) where p is the projection on the first factor. Definition 2. A principal bundle P (M, G, π) consists of a manifold P, a Lie group G, and a free right action of G on P denoted (u, g) '→ ug, such that the orbits of the action coincide with the fibres of the surjective submersion π : P → M, and such that M is covered by the domains of local sections σ: U → P, U ⊆ M, of π. Definition 3. A Lie group bundle, or LGB, is a smooth fibre bundle (K, q, M) in which each fibre (Km = q−1(m), and the fibre type G, has a Lie group structure, and for which there is an atlas {ψi: Ui × G → KUi } such that each {ψi,m : G → Km}, is an isomorphism of Lie groups. A morphism of LGB from (K, q, M) to (K′, q′, M′) is a morphism (F, f) of fibre bundles such that each Fm: Km → K′ is a morphism of Lie groups. In this paper, we will be discussing the Lie groupoid bundles. Here it is seen that to a Lie groupoid Ω on base B there is associated a collection of principal bundles Ωx(B, Ωx), all of which are mutually isomorphic and conversely, associated to any principal bundle P (B, G, p) there is a groupoid called the Ehresmann groupoid which is easily seen to be Lie. Further, some interesting properties of the category of Lie groupoids and bundles will be explored.

Keywords : groupoid, lie group, lie groupoid, bundle
Conference Title : ICLAA 2023 : International Conference on Linear Algebra and Applications
Conference Location : Dubai, United Arab Emirates
Conference Dates : December 25-26, 2023