

Closed Forms of Trigonometric Series Interm of Riemann's ζ Function and Dirichlet η , λ , β Functions or the Hurwitz Zeta Function and Harmonic Numbers

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Abstract : We present the results concerned with trigonometric series that include sine and cosine functions with a parameter appearing in the denominator. We derive two types of closed-form formulas for trigonometric series. At first, for some integer values, as we know that Riemann's ζ function and Dirichlet η , λ equal zero at negative even integers, whereas Dirichlet's β function equals zero at negative odd integers, after a certain number of members, the rest of the series vanishes. Thus, a trigonometric series becomes a polynomial with coefficients involving Riemann's ζ function and Dirichlet η , λ , β functions. On the other hand, in some cases, one cannot immediately replace the parameter with any positive integer because we shall encounter singularities. So it is necessary to take a limit, so in the process, we apply L'Hospital's rule and, after a series of rearrangements, we bring a trigonometric series to a form suitable for the application of Choi-Srivastava's theorem dealing with Hurwitz's zeta function and Harmonic numbers. In this way, we express a trigonometric series as a polynomial over Hurwitz's zeta function derivative.

Keywords : Dirichlet eta lambda beta functions, Riemann's zeta function, Hurwitz zeta function, Harmonic numbers

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